IMPLIES

February 14, 2014 IMPLIES.1

IMPLIES

The value of (P IMPLIES Q) is **F** iff
P is **T** and Q is **F**.

Truth Table for IMPLIES

P	Q	P IMPLIES Q
T	T	T
T	F	F
F	T	T
F	F	T

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A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to
reach the false conclusion

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A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to
reach the false conclusion

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1

A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion from the false hypothesis.

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A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion from the false hypothesis.

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A True Implication

(1=-1) IMPLIES (I am Pope)
The *whole implication* is true, even though both conclusion & hypothesis are false.

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Proving Validity

Instead of truth tables, can try to prove valid formulas symbolically using axioms and deduction rules

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2

modus ponens **rule**

antecedents

$$P, \quad P \text{ IMPLIES } Q$$
$$Q$$
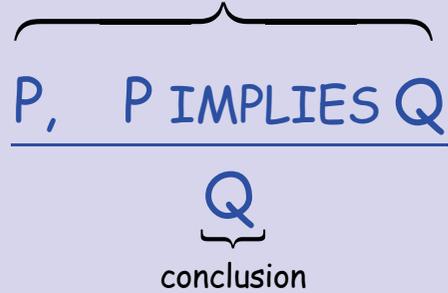
conclusion

Soundness

A sound rule preserves truth: if all the antecedents are true in some environment, then so is the conclusion.

Soundness

modus ponens is sound:
if P is true,
and P IMPLIES Q is true,
then Q must be true
—by truth table.

Soundness & Validity

Lemma: A rule is sound iff
AND{Antecedents}
    IMPLIES Conclusion
is valid.

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