Lecture 14: Baseball Elimination

In this lecture, we will be studying how to use max-flow algorithms to compute elimination of sports teams in a league. Concretely, let us consider the standings of the AL Eastern Division of the Major League Baseball on August 30, 1996[1], without Detroit's record.

Table 1: Standings of AL East on August 30, 1996.

Team	Wins (w_i)	Losses (ℓ_i)	To Play (r_i)	Games against each other				
NY	75	59	28	–	5	7	4	3
Baltimore	71	63	28	5	–	2	4	4
Boston	69	65	28	7	2	–	4	0
Toronto	63	71	28	4	4	4	–	0
Detroit	?	?	28	3	4	0	0	–
				NY	Ba	Bo	T	D

From this chart, how can we figure out if a team is eliminated? A naive sports writer can only compute that Team i is eliminated if $w_i + r_i < w_j$ for some other j. For example, if Detroit's record was $w_5 = 46$, then Detroit is certainly eliminated since $w_5 + r_5 = 46 + 28 < 75 = w_1$.

This condition, however, is sufficient, but not necessary. For instance, consider $w_5 = 47$. Though $w_5 + r_5 = 75$, either NY or Baltimore will reach 76 wins since they have 5 games left against each other. How can we determine if Detroit is eliminated for arbitrary values of w_5?

To answer this question, we can use max-flow. Consider the Figure 1, where capacity between s and node $i - j$ is the number of games left to be played between team i and j, between node $i - j$ and node $k = 1, 2, 3, 4$ is infinity, and node k and t is $w_5 + r_5 - w_k$. The intuition for the construction of the graph is that we will assume Detroit win all r_5 games, and try to keep the number of wins per team to be less than or equal to the total possible wins of Detroit ($\leq w_5 + r_5$).

Theorem 1. *Team 5 (Detroit) is eliminated if and only if max-flow does not saturate all edges leaving the source, i.e., max flow value* < 26.

Proof. Saturation of the edge capacity corresponds to playing all the remaining games. If all the games *cannot* be played, while keeping the total number of wins of a team to be less than or equal to $w_5 + r_5$, then Team 5 is eliminated. $\square$

[1]roughly speaking!

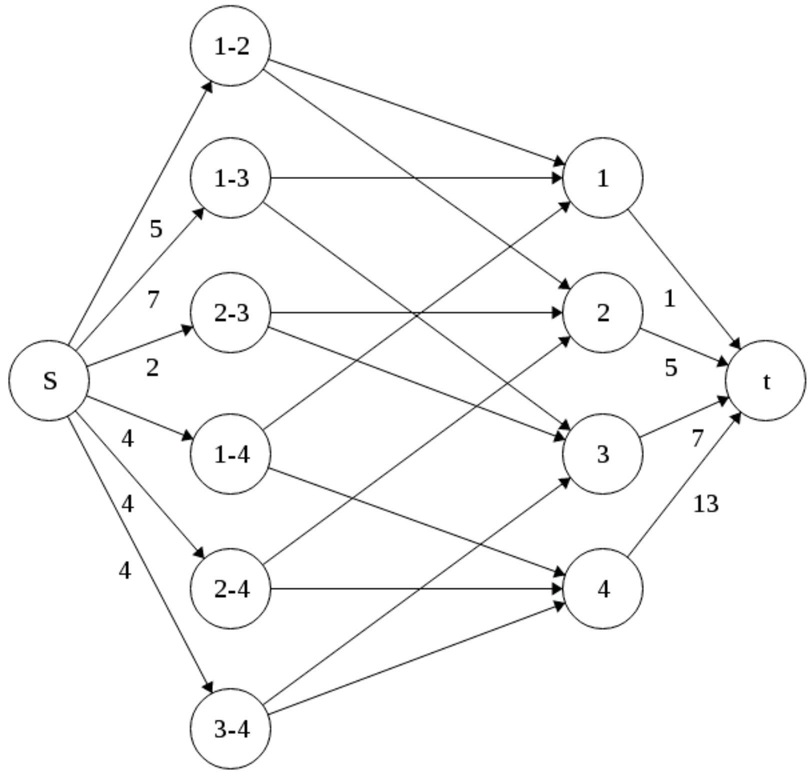

Figure 1: Flow network to determine if Team 5 is eliminated

In Figure 1, the min-cut (S, T) is $S = \{s, 1-2, 1-3, 2-3, 1, 2, 3\}$ and $T = \{1-4, 2-4, 3-4, 4, t\}$. The capacity of the min-cut $c(S, T) = 4+4+4+1+5+7$ $= 25 < 26$. Therefore, Team 5 Detroit is eliminated.

Alternate explanation: Note that the max-flow will find the subset of teams that eliminates Team 5. In this example, consider subset of teams 1,2, and 3. The total number of wins among the 3 teams is 215 wins, and they have 14 games left to play with each other. Then there will be 229 total wins at the end of the regular season. This implies that there exists at least one team that wins $\lceil \frac{229}{3} \rceil = 77$ games. Therefore, if Detroit only has 48 wins, then it is certainly eliminated $(48 + 28 = 76 < 77)$. We can find such set using max-flow and max-flow min-cut theorem.

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