Operational Amplifier Circuits
Comparators and Positive Feedback

Comparators: Open Loop Configuration

The basic comparator circuit is an op-amp arranged in the open-loop configuration as shown on the circuit of Figure 1. The op-amp is characterized by an open-loop gain A and let's assume that the output voltage *Vo* can go all the way to V_{DD} and V_{EE}. The output voltage is given by

$$Vo = A(V_+ - V_-) \tag{1.1}$$

Where V_+ and V_- correspond to the voltages at the non-inverting and the inverting terminals respectively.

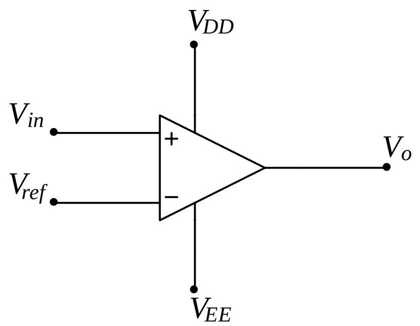


Figure 1. Basic non-inverting comparator.

For the circuit on Figure 1, $V_+ = V_{in}$ and $V_- = V_{ref}$. For $V_{ref} = 0$, the voltage transfer characteristic *Vo* versus *Vin* is as shown on Figure 2.

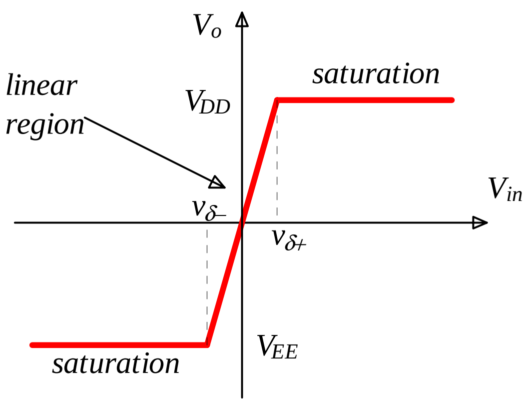


Figure 2. Voltage transfer characteristic of non-inverting comparator

When $V_{in} > v_{\delta+}$, $Vo = V_{DD}$ and for $V_{in} < v_{\delta-}$, $Vo = V_{EE}$.

The values $v_{\delta+}$ and $v_{\delta-}$ is inversely proportional to the open-loop gain A.

$$v_{\delta+} = \frac{V_{DD}}{A}$$
$$v_{\delta-} = \frac{V_{EE}}{A} \qquad (1.2)$$

Operation in the linear region is restricted to $v_{\delta-} < Vin < v_{\delta+}$. Outside this range the op-amp is driven to saturation.

For a practical op-amp A=200000 and for $V_{DD}=10V$ and $V_{EE}=-10V$, $v_{\delta+,-} = \pm 50\mu V$, a very small voltage. Therefore, the amplifier may be driven to saturation very easily.

For $V_{ref} > 0$, the voltage transfer characteristic *Vo* versus *Vin* is as shown on Figure 3.

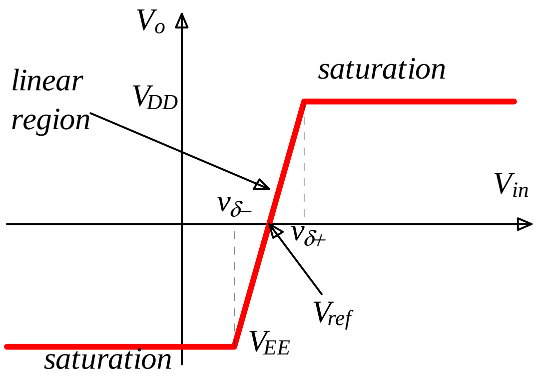


Figure 3. Voltage transfer characteristic of non-inverting comparator with non-zero reference

Here the characteristic is shifted to the right by V_{ref}. The range of Vin for operation in the linear region is the same and the saturation voltages are again V_{DD} and V_{EE}.

Due to the large value of A, the voltage range $[v_{\delta-}, v_{\delta+}]$ is very small and thus for the remainder of this treatment we will present the behavior of the comparator assuming infinite open-loop gain which corresponds to zero linear region. Therefore, the voltage transfer plot shown on Figure 3 will become as shown on Figure 4. Here we see a transition from one saturation region to the other as the voltage V_{in} crosses V_{ref}.

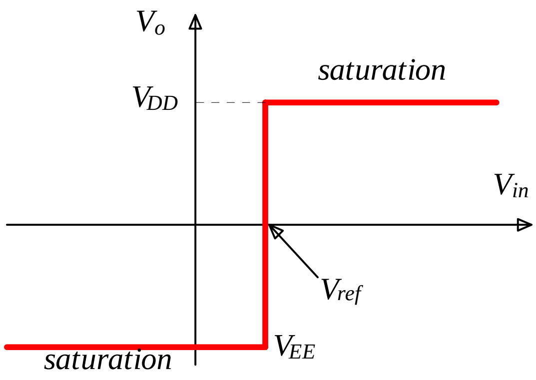


Figure 4. Ideal comparator

The comparator may also be arranged in the inverting configuration as shown on Figure 5 where the input signal is applied to the inverting terminal and the reference voltage is at the non-inverting terminal.

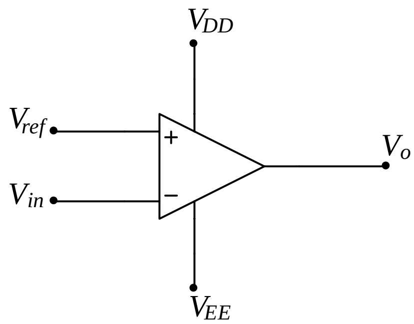


Figure 5. Inverting comparator

Since $Vo = A(V_{ref} - V_{in})$ the corresponding voltage transfer characteristic is shown on Figure 6.

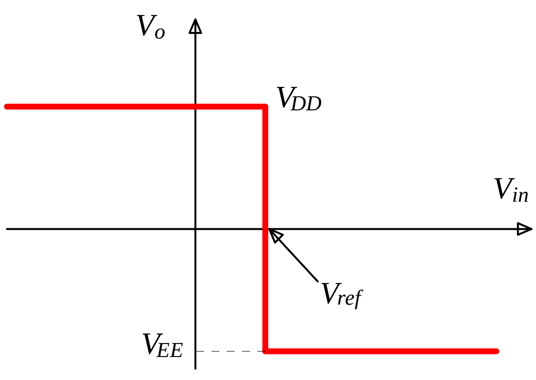


Figure 6

Another arrangement of an non-inverting comparator and its voltage transfer characteristic is shown on Figure 7(a).
The output voltage *Vo* is again

$$Vo = A(V_+ - V_-) \tag{1.3}$$

The voltage, V_+, at the non-inverting terminal is found by superposition ant it is

$$V_+ = V_{in}\frac{R_2}{R_1+R_2} + V_{ref}\frac{R_1}{R_1+R_2} \tag{1.4}$$

Since $V_- = 0$ the comparator will transition when V_+ crosses zero. This happens when

$$0 = V_{in(t)}R_2 + V_{ref}R_1 \tag{1.5}$$

And thus the transition voltage is

$$V_{in(t)} = -\frac{R_1}{R_2}V_{ref} \tag{1.6}$$

$V_+ > 0$ corresponds to $V_{in} > V_{in(t)}$ and the output goes positive (V_H). Similarly, $V_+ < 0$ corresponds to $V_{in} < V_{in(t)}$ and the output goes negative (V_L). The voltage transfer curve is shown on Figure 7(b).

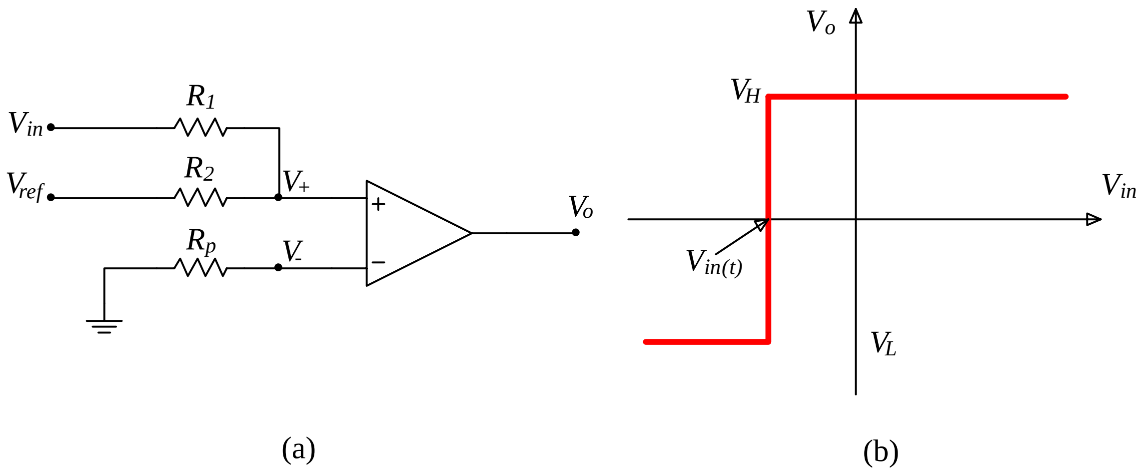


Figure 7. Non-inverting comparator circuit

The corresponding arrangement for the inverting configuration is shown on Figure 8.

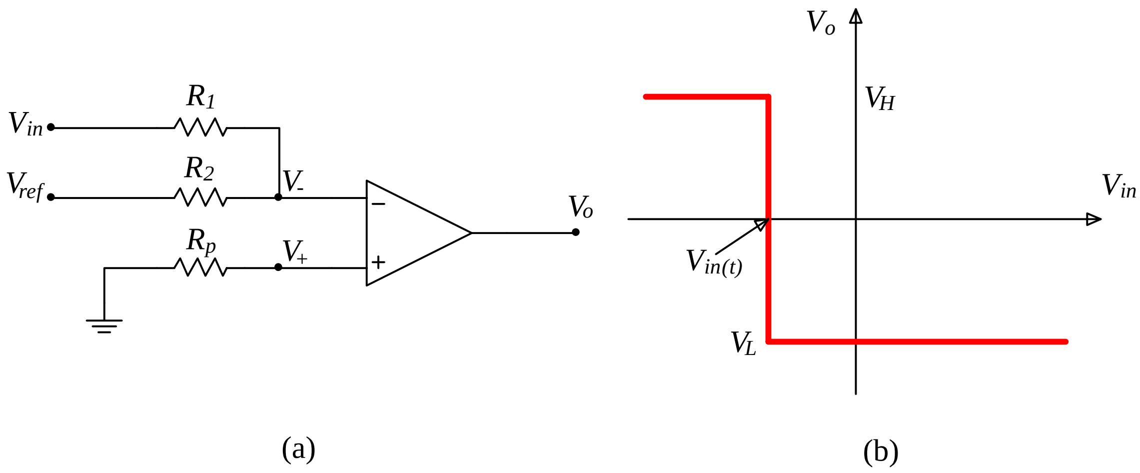


Figure 8. Inverting comparator and its voltage transfer characteristic.

A common application of a comparator circuit is in smoke alarm circuits as shown on Figure 9.

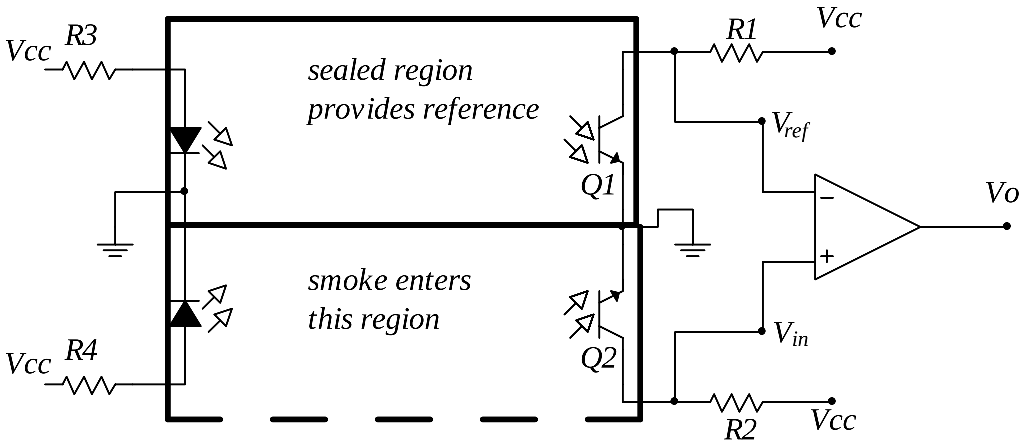


Figure 9. Smoke detector schematic

The diodes emit light which is detected by the phototransistors *Q1* and *Q2*. The top region is sealed and thus the operating point of transistor *Q1* does not change. This operating point is used as the reference for our comparator.

When smoke enters the lower region the operating point of phototransistor *Q2* changes thereby resulting in a change in the voltage V_{in} from the base (no smoke) value $V_{in(ns)}$.

The basic voltage characteristic of the comparator is as shown on Figure 10.

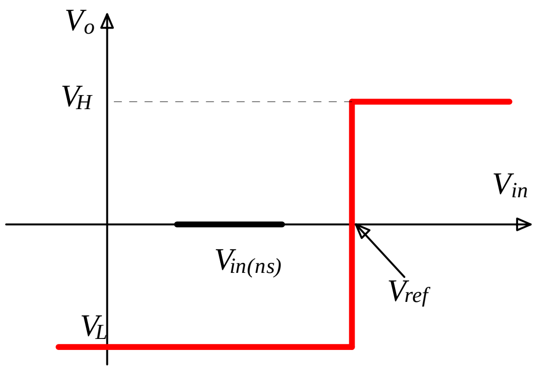


Figure 10

As the intensity of the light at the base of the phototransistor decreases due to smoke entering the region, the base current decreases and the voltage V_{in} will increase from the base (no smoke) value $V_{in(ns)}$. When the voltage V_{in} crosses V_{ref} the output of the comparator switches from V_L to V_H triggering the alarm.

Let's now consider what happens if the time evolution of the voltage V_{in} is as shown on Figure 11.

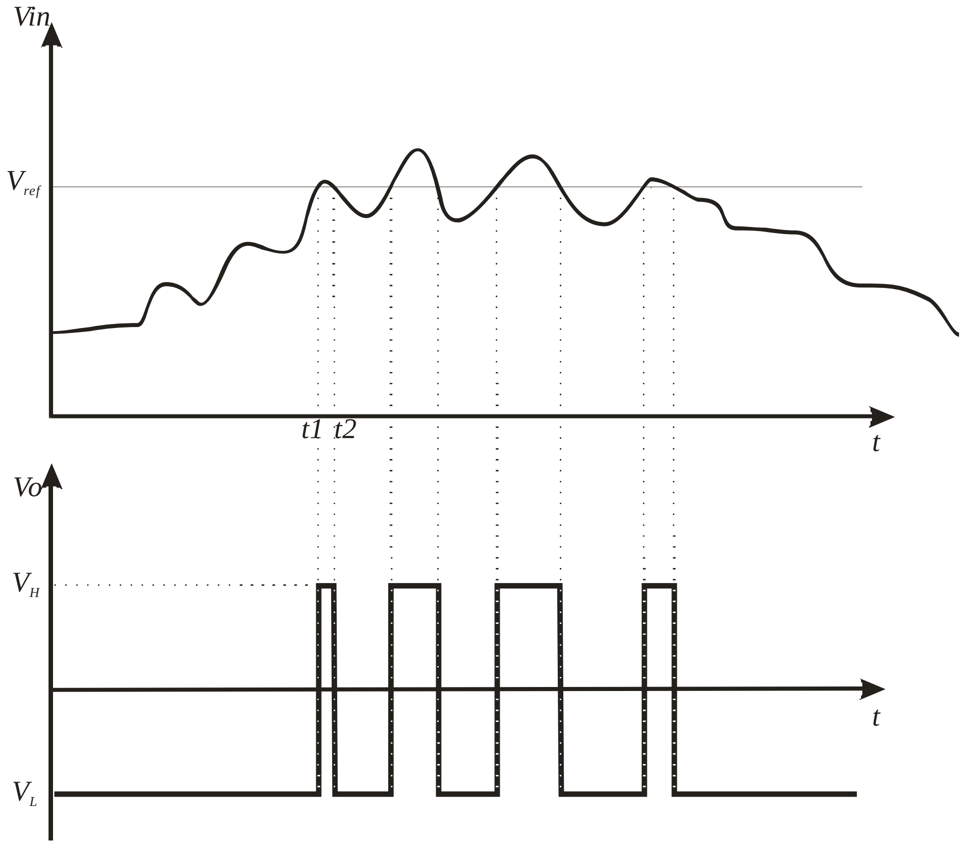


Figure 11

Note that as *Vin* exceeds *Vref*, the voltage at the output of the comparator switches from V_L to V_H and the alarm is turned on. Notice that sometime after t1 the voltage *Vin* begins to decrease and at time t2 is crosses *Vref* . Now the output of the comparator switches back to V_L and the alarm is turned off. This fluctuation in *Vin* might be noise in the signal or some temporary decreases in the smoke (the effect of a person fanning at the smoke detector). The smoke is still around however and as it is shown on the evolution, the transition voltage will be crossed numerous times turning the alarm on and off each time. This is an undesirable operation and we would like to modify the circuit in such a way that there is some "noise immunity" in the behavior of the circuit.

Positive Feedback. Schmitt Trigger

For example, if we are able to specify a voltage range within which the state of the output can be at either value then we should be able to build the appropriate level of noise immunity. The conceptual representation of the idea is shown on Figure 12. In this case the alarm turns on at time t1 and turns off at time t2 avoiding all the chatter in between.

The circuit that will accomplish this employs positive feedback and it is called Schmitt Trigger. The basic form of the circuit is shown on Figure 13. Positive feedback is achieved by connecting the feedback path to the non-inverting terminal of the op-amp.

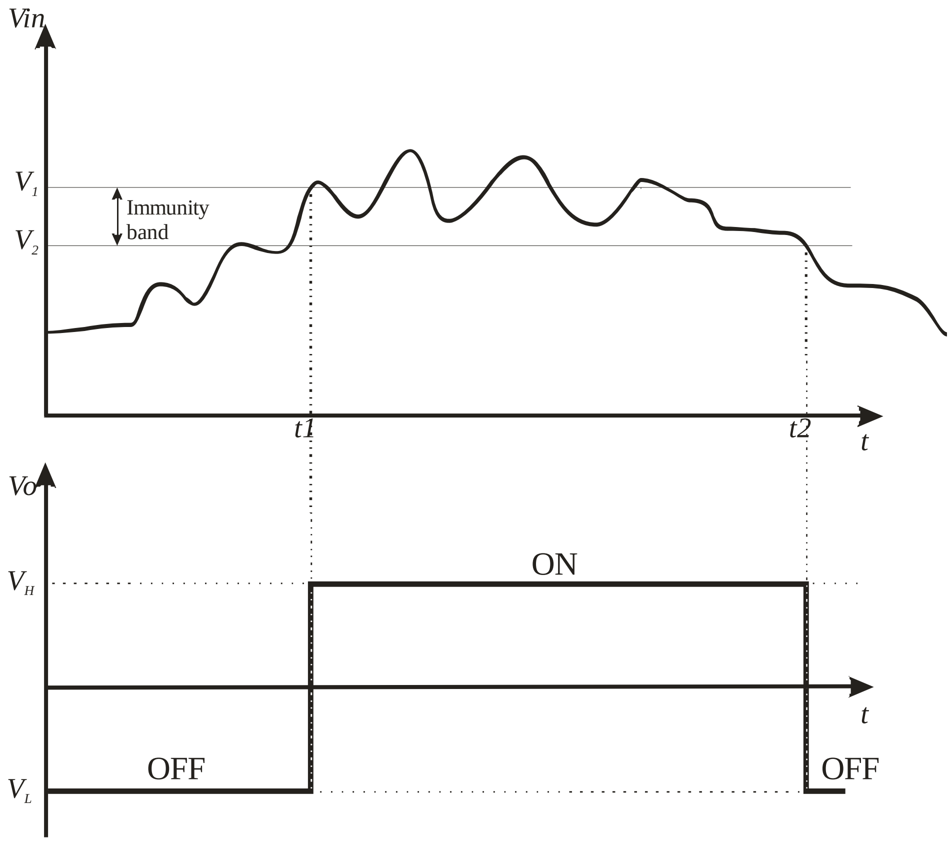


Figure 12

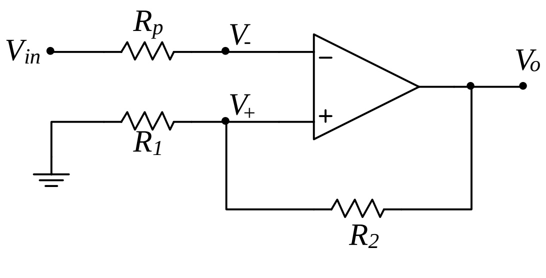


Figure 13. Inverting Schmitt trigger.

In our example, the input signal is applied to the inverting terminal and consequently, the circuit is called **Inverting Schmitt Trigger**.

The voltage V_+ is related to the output voltage V_o by the simple voltage divider formed by resistors R_1 and R_2.

$$V_+ = V_o \frac{R_1}{R_1 + R_2} \tag{1.7}$$

Note that V_+ depends on the output voltage V_o and the in turn V_o depends on the difference between V_+ and V_- : $V_o = A(V_+ - V_-)$

The key to analyzing these positive feedback circuits is to assume an initial condition. The possible values that V_o can take are V_H and V_L where $V_H > V_L$, also assume that $V_L < 0$.

By assuming that the initial state of V_o is V_H then,

$$V_+ = V_H \frac{R_1}{R_1 + R_2} \tag{1.8}$$

and the output is

$$V_o = A\left[V_H \frac{R_1}{R_1 + R_2} - V_{in}\right] \tag{1.9}$$

Therefore, as long as $V_{in} < V_H \frac{R_1}{R_1 + R_2}$ the output will remain at V_H.

The transition from V_H to V_L will happen when $V_{in} > V_H \frac{R_1}{R_1 + R_2}$. This part of the operation is shown on Figure 14(a) where the voltage $V_{TU} \equiv V_H \frac{R_1}{R_1 + R_2}$.

So when $V_{in} > V_{TU}$, $V_o = V_L$ and $V_+ = V_L \frac{R_1}{R_1 + R_2}$.

Now let's see what happens when V_{in} starts decreasing. In order for a transition to happen again the term $\left[V_L \frac{R_1}{R_1 + R_2} - V_{in}\right]$ has to change sign. This will happen when $V_{in} < V_L \frac{R_1}{R_1 + R_2}$ as shown on Figure 14(b) where $V_{TL} \equiv V_L \frac{R_1}{R_1 + R_2}$.

The complete voltage transfer characteristic is obtained by combining Figure 14(a) and (b) and it is shown on Figure 15. The voltage at which the state of the output changes depends on whether the input voltage increases or decreases. When V_{in} is between V_{TU} and V_{TL} the state of V_o can be either V_H or V_L - two possible states. For this reason this circuit is also called a **Bistable Multivibtator**. The voltage transfer characteristic shows that there is **Hysteresis Effect**. The width of the hysteresis region, $V_{TU} - V_{TL}$, corresponds to the noise immunity of the trigger circuit.

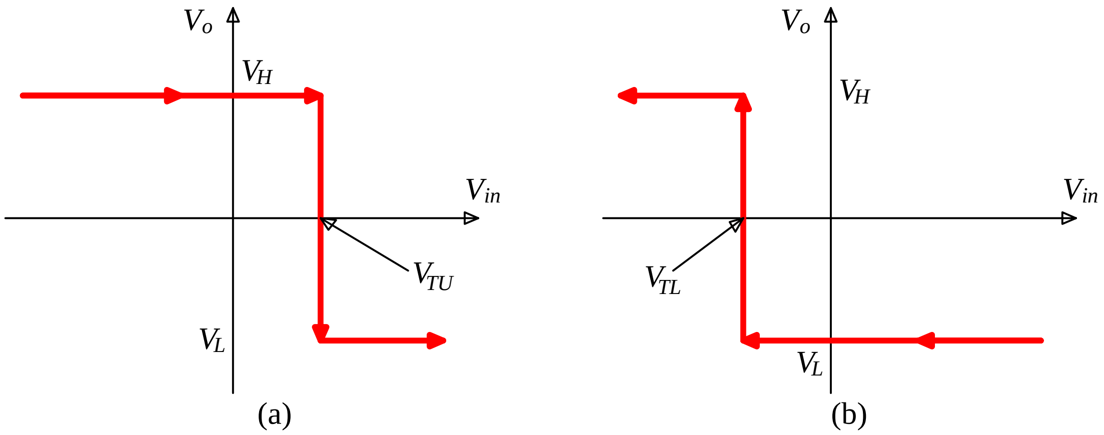


Figure 14.

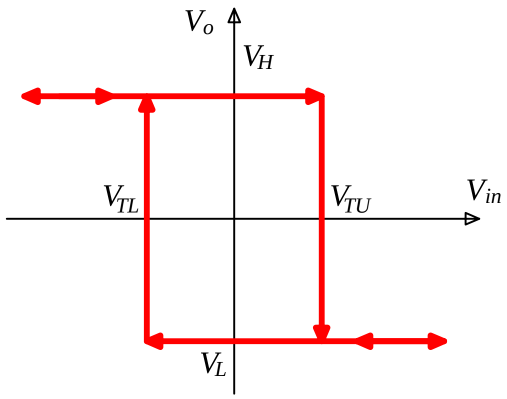


Figure 15. Voltage characteristic of inverting Schmitt trigger

Non-Inverting Schmitt Circuit

When the input signal is applied to the non-inverting terminals as in the circuit of Figure 16 the resulting circuit is the non-inverting Schmitt trigger.

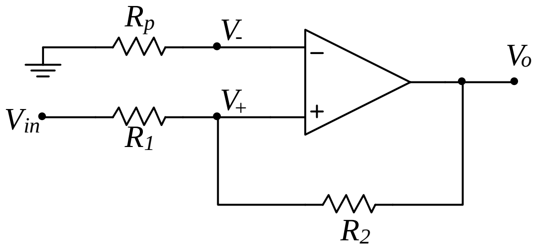


Figure 16. Non-inverting Schmitt trigger

The voltage V_+ at the non-inverting terminal is a combination of the output voltage, V_o, and the input voltage, V_{in}.

$$V_+ = V_{in}\frac{R_2}{R_1+R_2} + V_o\frac{R_1}{R_1+R_2} \tag{1.10}$$

Since $V_o = A(V_+ - V_-)$ the transitions will occur when V_+ crosses zero.

$$0 = V_T R_2 + V_o R_1 \tag{1.11}$$

Where V_T represents the transition voltage. Let's assume the initial state $V_o = V_L$. Then

$$V_T = -\frac{R_1}{R_2}V_L \tag{1.12}$$

And since $V_L < 0$, V_T is positive and we call it V_{TU}. This corresponding part of the characteristic plot is shown on
Figure **17**(a).

Now the output is at V_H and will stay at this state until the voltage V_+ crosses zero. This will happen at the transition voltage V_T which must satisfy

$$0 = V_T R_2 + V_H R_1 \tag{1.13}$$

This is the lower transition voltage V_{TL} given by

$$V_{TL} = -\frac{R_1}{R_2} V_H \tag{1.14}$$

For $V_H > 0$, $V_{TL} < 0$ and the corresponding path and transition voltage is shown on Figure 17(b). The complete voltage characteristic is shown on Figure 18.

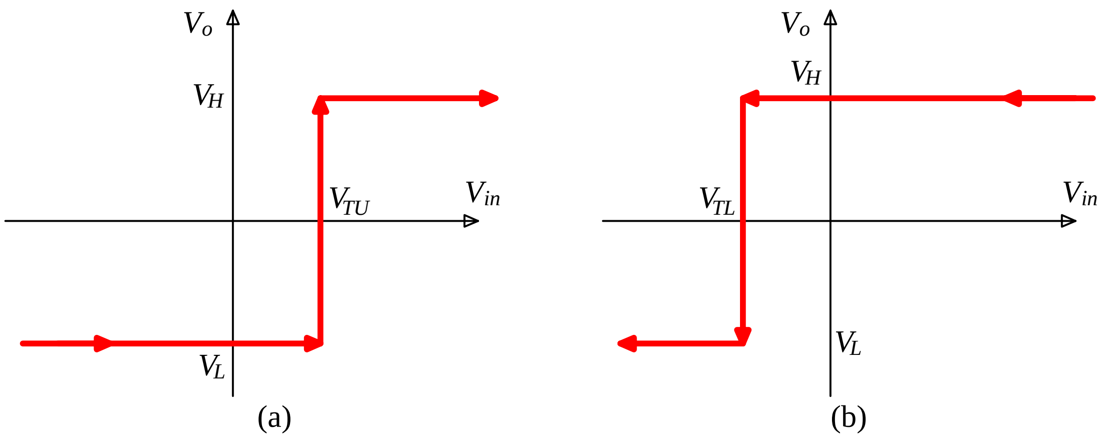


Figure 17.

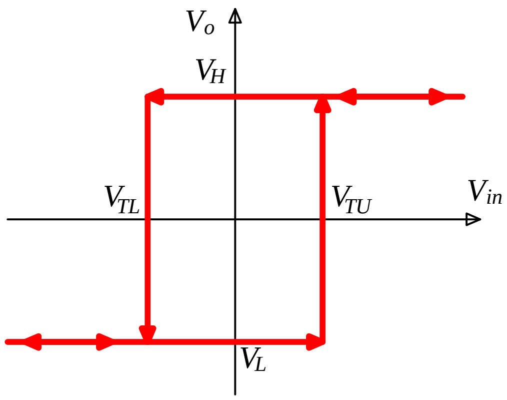


Figure 18

Schmitt Trigger with Reference Voltage

The circuit on Figure 19(a) is an inverting Schmitt trigger with a reference voltage V_{ref}. The analysis of the circuit proceeds as before.

The voltage V_+ at the non-inverting terminal is a combination of the output voltage, V_o, and the reference voltage, V_{ref}.

$$V_+ = V_{ref}\frac{R_2}{R_1+R_2} + V_o\frac{R_1}{R_1+R_2} \tag{1.15}$$

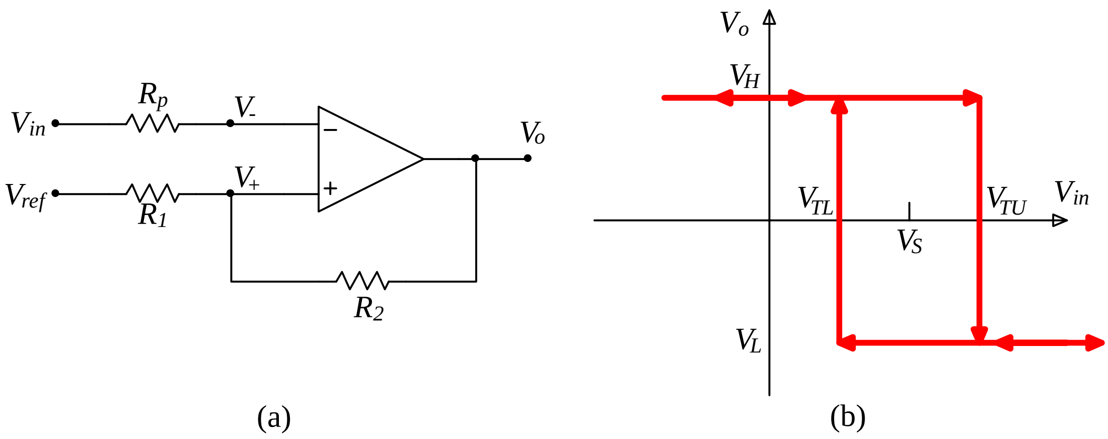


Figure 19. Inverting Schmitt trigger with reference voltage. (a) circuit, (b) voltage transfer characteristic

By comparing Equation (1.15) to Equation (1.8) we see that the location of the transition voltages is simply shifted by $V_S = V_{ref}\frac{R_2}{R_1+R_2}$.

In this case the transition voltages are:

$$V_{TU} = V_S + V_H\frac{R_1}{R_1+R_2} \tag{1.16}$$

$$V_{TL} = V_S + V_L\frac{R_1}{R_1+R_2} \tag{1.17}$$

The voltage transfer curve of the inverting Schmitt trigger with reference voltage is shown on Figure 19(b).

A non-inverting Schmitt trigger circuit with reference voltage is shown on Figure 20(a) with the corresponding voltage characteristic on Figure 20(b). The transition voltages are given by:

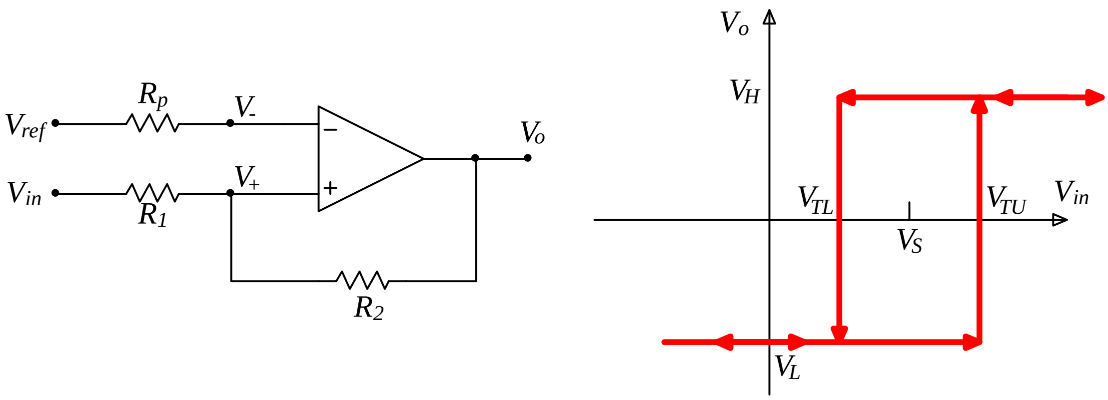
$$V_{TU} = V_S - V_L \frac{R_1}{R_2} \tag{1.18}$$

$$V_{TL} = V_S - V_H \frac{R_1}{R_2} \tag{1.19}$$

Where

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{ref} \tag{1.20}$$

Therefore, with the appropriate choice of *Vref*, *R1* and *R2* we may design a Schmitt trigger with the desired hysteresis characteristics.



(a) (b)

Figure 20. Non-Inverting Schmitt trigger with reference voltage. (a) circuit, (b) voltage transfer characteristic

Schmitt Trigger Oscillator: Astable Multivibrator

By combining negative and positive feedback and a capacitor for energy storage we can design a circuit that generates a square wave output. The circuit is shown on Figure 21.

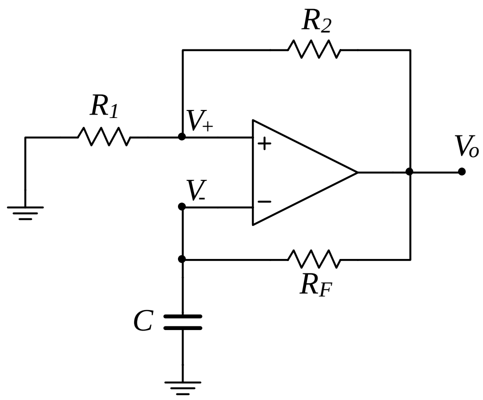


Figure 21. Astable Multivibrator

Let's analyze the operation of this circuit for $R_1 = R_2 = R$. Furthermore let's define the saturation voltages V_H and V_L which we take to be symmetric about zero: $V_H = -V_L$.

In order to proceed we must assume an initial state for the output voltage V_o. Let's start with $V_o = V_L$ which gives

$$V_+ = \frac{1}{2}V_L \qquad (1.21)$$

Recall that we are operating with the positive feedback and thus the output is given by

$$V_o = A(V_+ - V_-) \qquad (1.22)$$

The voltage $V_+ = \frac{1}{2}V_L$ is determined by the characteristics of the capacitor C and the resistor R_F. The form of V_- is the standard exponential function of a transient RC circuit.

The voltage V_o will change state (i.e. switch from $V_o = V_L$ to $V_o = V_H$) when V_- becomes less than $\frac{1}{2}V_L$.

The general form of the voltage V_- is given by

$$V_- = V_{final} + (V_{initial} - V_{final})e^{-t/R_F C} \qquad (1.23)$$

The evolution after the first transition (V_o going from V_L *to* V_H) which happens when $V_- = \frac{1}{2}V_L$ is found from Equation (1.23) with $V_{initial} = \frac{1}{2}V_L$ and $V_{final} = V_H$

$$V_- = V_H + (\frac{1}{2}V_L - V_H)e^{-t/R_F C} \qquad (1.24)$$

V_- starts to increase exponentially towards V_H. The function that describes the relationship between the inputs and the output of the op-amp is $V_o = A(\frac{1}{2}V_H - V_-)$ and when V_- crosses $\frac{1}{2}V_H$ the output will switch from V_H to V_L and the process continuous

The time evolution of the voltages of interest are shown on Figure 22 where $V_H = +5V$, $V_L = -5V$ and $R_1 = R_2$. Assuming that the initially $V_o = V_H = +5V$ and $V_- = \frac{1}{2}V_L = -\frac{5}{2}Volts$, the equation that describes the evolution of V_- for *t>0* is

$$V_- = V_H - \frac{3}{2}V_H e^{-t/R_F C} \qquad (1.25)$$

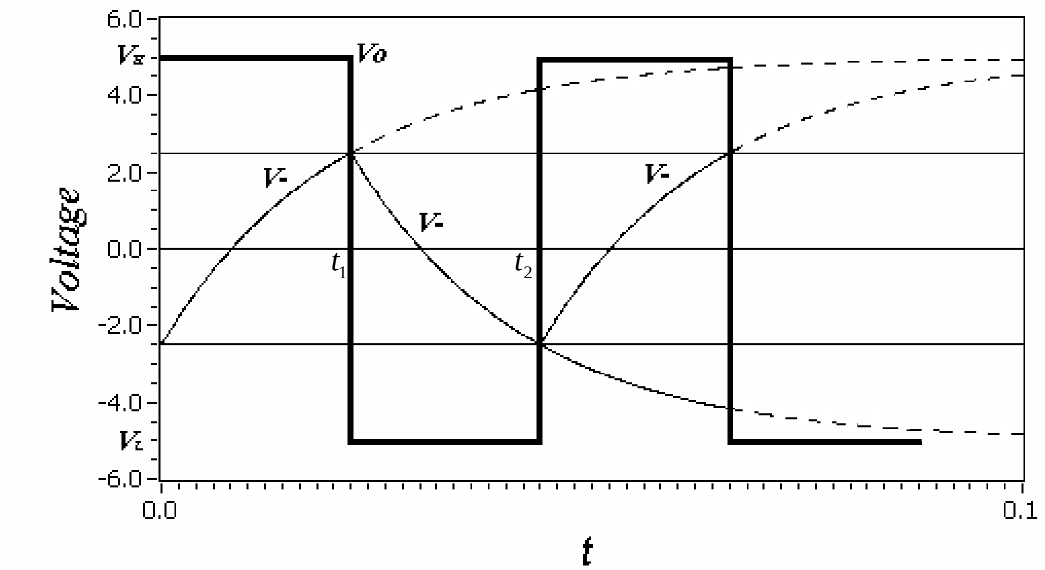


Figure 22. Oscillator voltage characteristics

Since $V_o = 5Volts$, $V_+ = \frac{5}{2}Volts$ and the output will change state once V_- crosses $\frac{5}{2}Volts$. This happens at $t = t_1$ and t_1 satisfies

$$\frac{5}{2} = 5 - \frac{3}{2}5e^{-t_1/R_F C} \tag{1.26}$$

And solving for t_1 we obtain

$$t_1 = R_F C \ln(3) \tag{1.27}$$

Due to the symmetry of the problem ($V_H = -V_L$) the time between t_1 and t_2 is also given by

$$t_2 - t_1 = R_F C \ln(3) \tag{1.28}$$

And so the period of the square wave is

$$\boxed{T = 2R_F C \ln(3)} \tag{1.29}$$

An additional consequence of the frequency is that the duty cycle[1] of this square wave is 50%. If a higher (>50%) duty cycle is desired then $|V_H| > |V_L|$

The general expression for the period of the square wave is

$$T = 2R_F C \ln\left(1 + 2\frac{R1}{R2}\right) \tag{1.30}$$

The proof of Equation (1.30) follows from the analysis above and it is left as an exercise for the student.

Note that the period of the square wave does not depend on the op-amp characteristics but only on the external components used in building the circuit. The quality of the square wave as the frequency increases is however dependent on the frequency characteristics of the op-amp.

[1] $duty\ cycle = \frac{t_1}{t_2}$