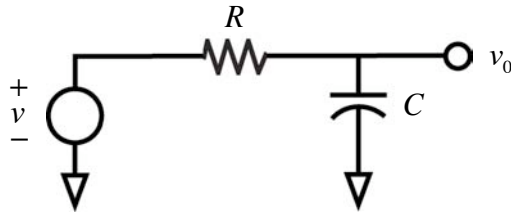# 6.301 Solid State Circuits
Recitation 5: Miller Effect
Prof. Joel L. Dawson

With regards to high-frequency amplifiers, or high-speed circuits in general, capacitance is bad news. One way to think about this is to realize that with "high-speed" circuits, we're often talking about how to make the voltage at a given node change rapidly:

That is, we want $\left|\frac{dv_0}{dt}\right|$ to be large. What keeps this from happening? Writing *KCL*:

$$\frac{v - v_0}{R} = c\frac{dv_0}{dt} \Rightarrow \left|\frac{dv_0}{dt}\right| = \frac{|v - v_0|}{RC}$$

Our options are:

1) Increase $|v - v_0|$
2) Decrease R
3) Decrease C

In general, we can't use (1) as a good design strategy because large $|v - v_0|$ all the time implies significant attention between v and v_0. That leaves us often managing the RC product whichever way we can.

How do we "manage" the RC product? Minimize R, minimize C, of course. But when there is gain between various nodes in a circuit, managing capacitance also means that we make sure the Miller effect doesn't hurt us.

CLASS EXERCISE: Miller Effect

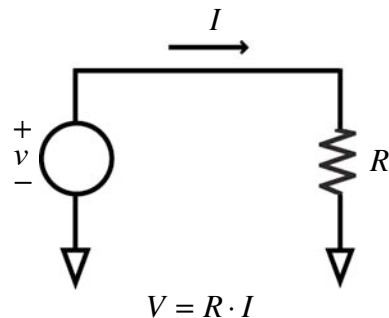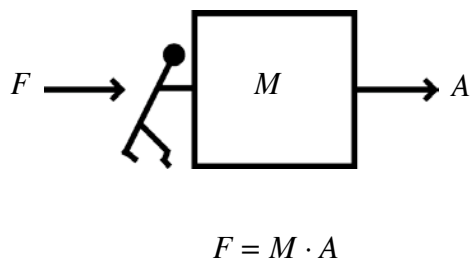1) Compute the input impedance for the following circuit:
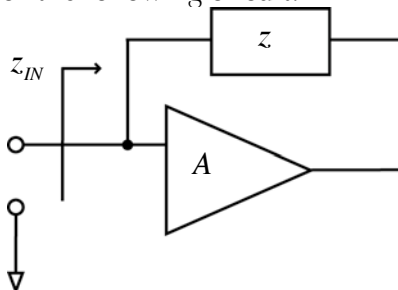
2) Simplify your result for the case of $A = +1$.

(Workspace)

There's a mechanical analog to the Miller effect. Suppose you came upon a large block sitting on a frictionless surface. One way to figure out its mass would be to give it a well-defined push and then observe its acceleration.

$$F = M \cdot A$$

$$V = R \cdot I$$

Now suppose that unbeknownst to you, someone on the other side knew, at all times, what force you would apply, and used that information to apply a force himself. It would distort your perception of the mass of the block!
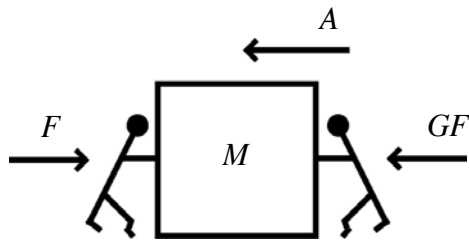

$$2F = M \cdot A$$

$$F = \left(\frac{1}{2} M \right) A$$
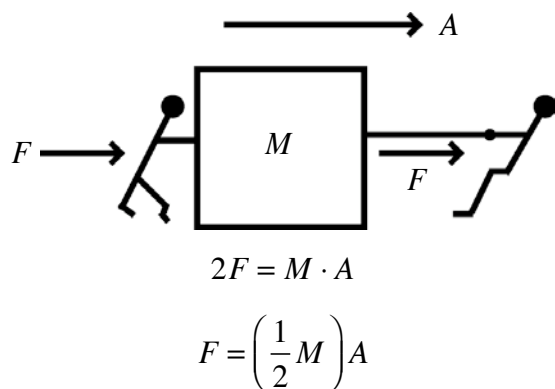
Mass seems smaller!

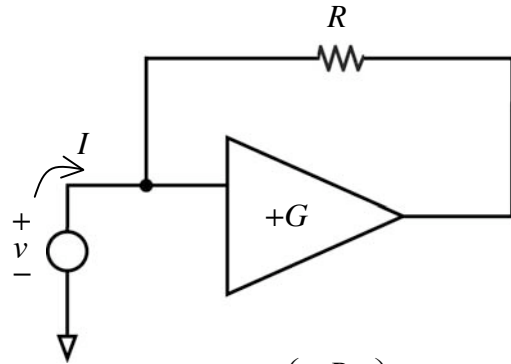$$(1 + G)V = RI \rightarrow V = \left(\frac{R}{1 + G} \right) I$$
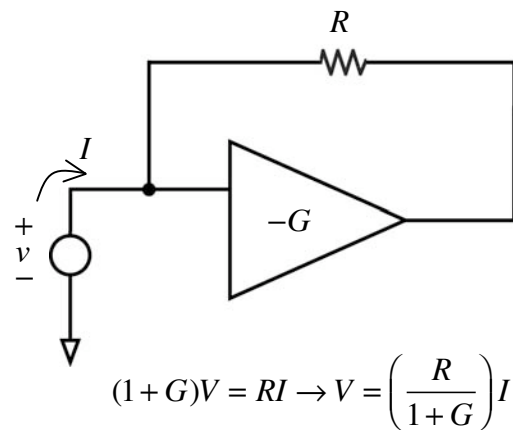
Resistance seems smaller!
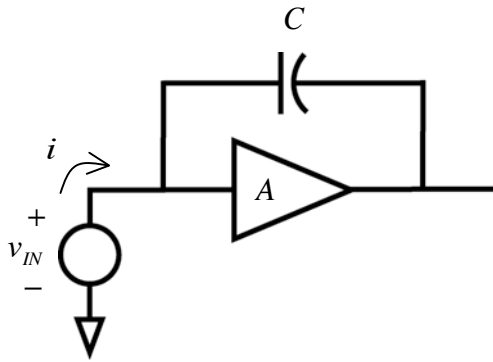
$$(1-G)F = MA$$

$$F = \left(\frac{M}{1-G}\right)A$$

Mass seems negative!

$$V = \left(\frac{R}{1-G}\right)I$$

Resistance seems negative!

In circuits, the Miller effect can have a significant impact on the bandwidth of our final designs. First, let's specify our result for capacitors.

$$i = \frac{v_{IN}(1-A)}{\frac{1}{Cs}}$$

$$\frac{v_{IN}}{i} = \frac{1}{(1-A)Cs}$$

So if A is negative, our voltage source "sees" a capacitance that is much larger than C.

Where has this Miller effect shown up in our circuits?
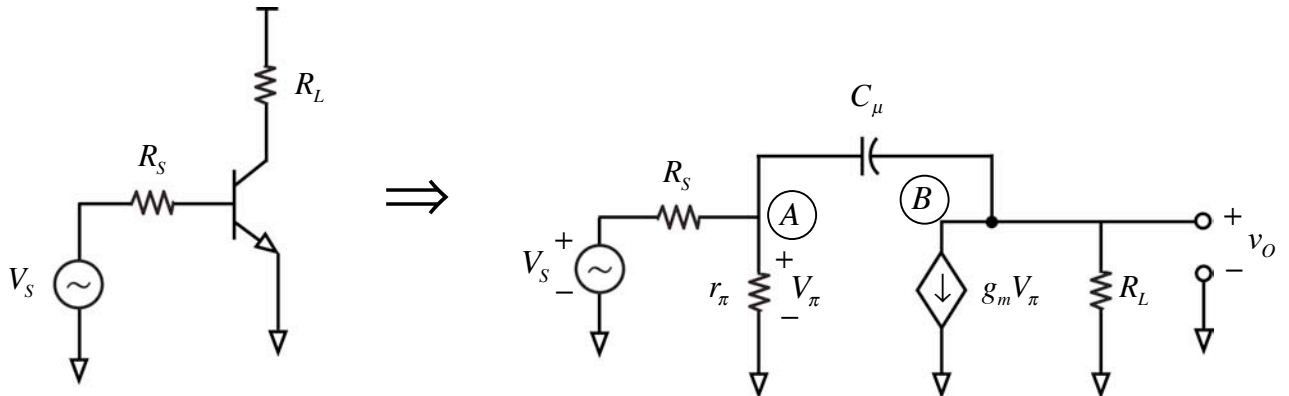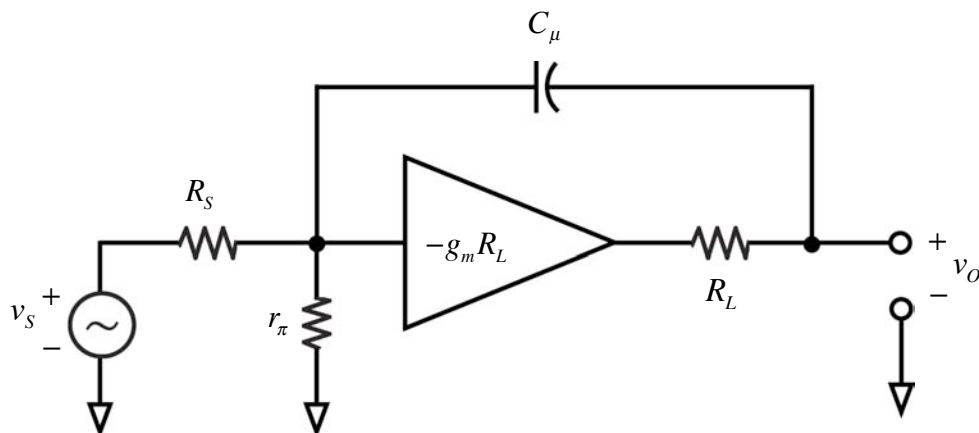
1) <u>Common Emitter Amplifier</u>

<u>Significant</u> negative gain between nodes (A) and (B):

Qualitatively, we can look at this diagram and sense trouble.

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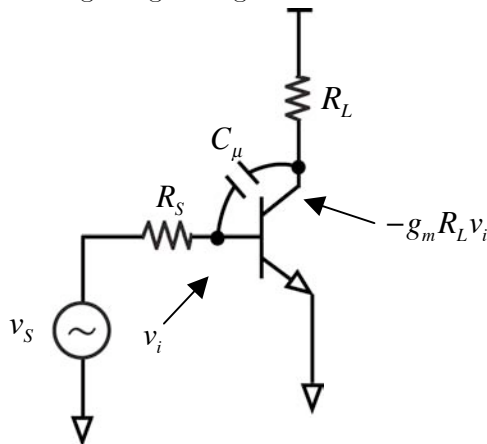
Recitation 5: Miller Effect

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Analytically, the Miller effect shows up in the OCT Calculation:

$$\tau_{\mu 0} = R_{\mu 0} C_\mu = \left(R_s \| r_\pi + R_L + \underbrace{g_m R_L \left(R_s \| r_\pi \right)} \right) C_\mu$$
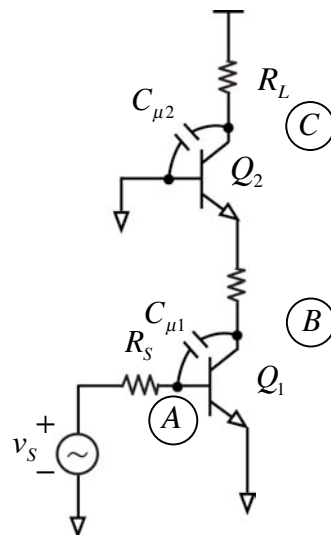
Amplifier gain magnifies the OCT contribution

2) Cascode Amplifier Stage

The big drawback with the common emitter stage is that we have a capacitance, C_μ, connected across a large negative gain:

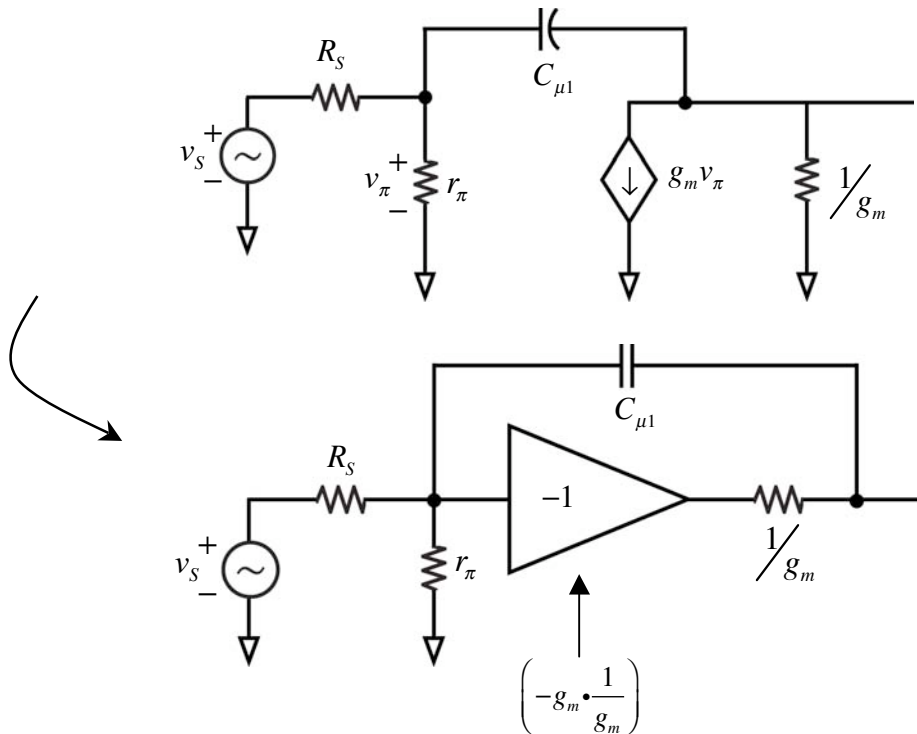How does a cascode help to mitigate this? By "managing" the Miller effect!


Let's find the gain between points (A) and (B)...

Recall that the impedance looking into the emitter of Q_2 is $\sim \frac{1}{g_m}$. For Q_1, then, we can draw the following small-signal model:

The reason the cascode is a speed improvement is that it separates the two nodes between which there is significant negative gain. There is no single capacitor that bridges the two nodes.

We actually saw something like this with the µA733 video amplifier. A differential half-circuit might look something like the following (consider the case of no degeneration):

First Stage

Second Stage
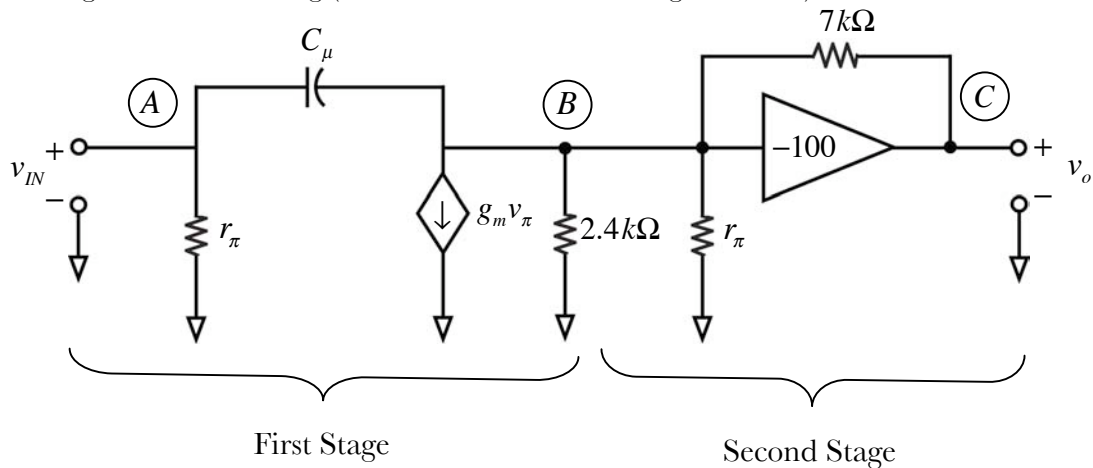
Node (A) is the input node, node (B) is the virtual ground, and node (C) is the output. Notice how here, as with the cascode, we have managed to avoid having a C_μ bridge two nodes between which there is a high negative gain.

Summary:

1) Reduce Rs as much as possible.
2) Reduce Cs as much as possible.
3) Try not to let Miller effect reduce the bandwidth.
4) Sometimes, use Miller effect to improve bandwidth.
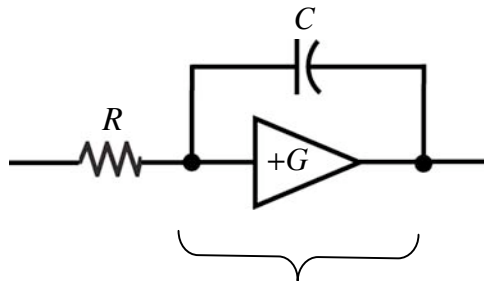
Little bit of negative capacitance! Use with care.

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