6.334 Power Electronics
Spring 2007

Chapter 3

Power Factor and Measures of Distortion

Read Chapter 3 of "Principles of Power Electronics" (KSV) by J. G. Kassakian, M. F. Schlecht, and G. C. Verghese, Addison-Wesley, 1991. Look at the AC side.

Definitions and Identities

Two functions X and Y are orthogonal over $[a, b]$ if:

$$\int_a^b X(t)Y(t)dt = 0 \tag{3.1}$$

Now:

- $\int_0^{2\pi} \sin(m\omega t)\sin(n\omega t + \phi)d\omega t = 0$, if $n \neq m \Rightarrow$ sinusoids of different frequencies are orthogonal.

- $\int_0^{2\pi} \sin(\omega t)\cos(\omega t) d\omega t = 0 \Rightarrow$ sine and cosine are orthogonal.

In general:

$$\frac{1}{2\pi}\int_0^{2\pi} \sin(\omega t)\sin(\omega t+\phi) = \frac{1}{2}\cos\phi \tag{3.2}$$

These definitions will be useful for calculating power, etc.

Suppose we plug a resistor into the wall.

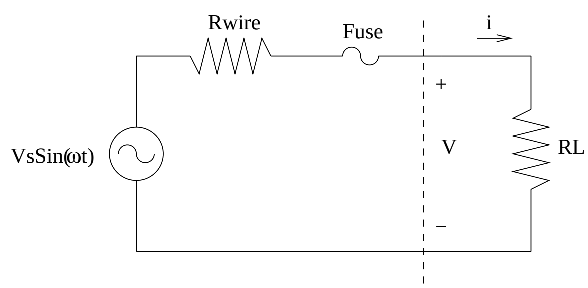

Figure 3.1: Resistor

$$\begin{aligned} P &= < Vi > \\ &= V_{RMS} i_{RMS} \\ &= i_{RMS}^2 R \end{aligned} \tag{3.3}$$

The fuse is rated for a specific RMS current. Above that, it will blow so that dissipation in R_{wire} does not start a fire. Neglecting R_{wire}, for $115V_{AC,RMS}$, $15A_{RMS}$ fuse, we get ~ 1.7kW max from wall.

Suppose instead we plug an inductor into the wall.

Neglecting R_{wire}:

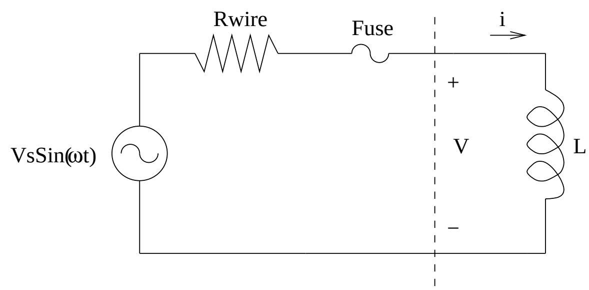

Figure 3.2: Inductor

$$i = -\frac{V_s}{\omega L}\cos(\omega t) \tag{3.4}$$

$$\begin{aligned} <P> &= \frac{1}{2\pi}\int V(t)i(t)d(\omega t) \\ &= -\frac{V_s^2}{2\pi\omega L}\int \sin(\omega t)\cos(\omega t)d(\omega t) \\ &= 0 \text{ (of course)} \end{aligned} \tag{3.5}$$

Mathematically, it is because V and i are orthogonal. While we draw no real power, we still draw current.

$$\begin{aligned} i_{RMS} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)d(\omega t)} \\ &= \frac{V_s}{\sqrt{2}\omega L} \end{aligned} \tag{3.6}$$

$$@115V, 60\text{Hz}, L \leq 20\text{mH} \rightarrow i_{RMS} \geq 15\text{A} \tag{3.7}$$

So we still will blow the fuse (to protect the wall wiring), even though we do not draw any real power at the output! (some power dissipated in R_{wire}). In this case we are not utilizing the source well.

Power Factor

To provide a measure of the utilization of the source we define Power Factor.

$$P.F. \doteq \frac{< P >}{V_{RMS} i_{RMS}} = \frac{\text{Real Power}}{\text{Apparent Power}} \tag{3.8}$$

For a resistor $< P >= V_{RMS} i_{RMS} \rightarrow P.F. = 1$ best utilization. For a inductor $< P >= 0 \rightarrow P.F. = 0$ worst utilization.

Consider a rectifier drawing some current waveform,

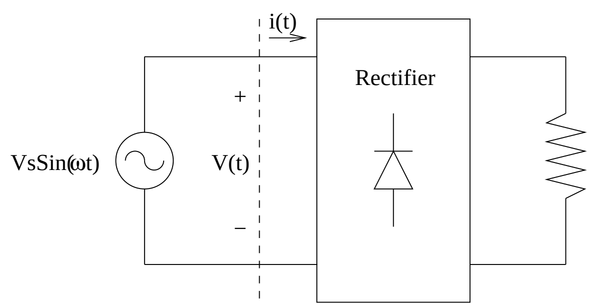

Figure 3.3: Rectifier

Express $i(t)$ as a Fourier series:

$$\begin{aligned} i(t) &= \sum_{n=0}^{\infty} i_n \sin(n\omega t + \phi_n) \text{ Sum of weighted shifted sinusoids} \quad (3.9) \\ \text{Note: } i_{RMS} &= \sqrt{\frac{1}{2} i_1^2 + \frac{1}{2} i_2^2 + \cdots + \frac{1}{2} i_n^2 + \cdots} \\ < P > &= \frac{1}{2\pi} \int_{2\pi} V(t) i(t) d(\omega t) \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2\pi}\int_{2\pi} V_s \sin(\omega t) \sum_n i_n \sin(n\omega t + \phi_n) \\
&= \sum_{n=0}^{\infty} \frac{1}{2}\int_{2\pi} V_s i_n \sin(\omega t)\sin(n\omega t + \phi_n) \qquad (3.10)
\end{aligned}
$$

By orthogonality all terms except fundamental drop out.

$$
\begin{aligned}
< P > &= \frac{1}{2}\int_{2\pi} V_s i_1 \sin(\omega t)\sin(\omega t + \phi_1) \\
&= \frac{V_s i_1}{2}\cos\phi_1 \\
&= V_{s,RMS} i_{1,RMS} \cos\phi_1 \qquad (3.11)
\end{aligned}
$$

So the only current that contributes to real power is the fundamental component in phase with the voltage.

$$
\begin{aligned}
P.F. &= \frac{V_{RMS} i_{1,RMS}}{V_{RMS} i_{RMS}}\cos\phi_1 \\
&= \frac{i_{1,RMS}}{i_{RMS}}\cos\phi_1 \qquad (3.12)
\end{aligned}
$$

We can break down into two factors:

$$
\begin{aligned}
P.F. &= \left(\frac{i_{1,RMS}}{i_{RMS}}\right)\cdot\cos\phi_1 \\
&= k_d(\text{distortion factor})\cdot k_\theta(\text{displacement factor})
\end{aligned}
\qquad (3.13)
$$

- k_d, distortion factor (≤ 1) tells us how much the utilization of the source is reduced because of harmonic currents that do not contribute to power.

- k_θ, displacement factor (≤ 1) tells us how much utilization is reduced due to phase shift between the voltage and fundamental current.

Total Harmonic Distortion (THD)

Consider another measure of distortion: Total Harmonic Distortion (THD).

$$THD \doteq \sqrt{\frac{\sum_{n\neq 1} i_n^2}{i_1^2}} \tag{3.14}$$

This measure the RMS of the harmonics normalized to the RMS of the fundamental (square root of the power ratio). Distortion factor and THD are related:

$$\begin{aligned}
THD &= \sqrt{\frac{\sum_{n\neq 1} i_n^2}{i_1^2}} \\
&= \sqrt{\frac{i_{RMS}^2 - i_{1,RMS}^2}{i_{1,RMS}^2}} \\
THD^2 &= \frac{i_{RMS}^2}{i_{1,RMS}^2} - 1 \\
\frac{i_{RMS}^2}{i_{1,RMS}^2} &= 1 + THD^2 \\
\frac{i_{RMS}}{i_{1,RMS}} &= \sqrt{1 + THD^2} \\
k_d &= \sqrt{\frac{1}{1 + THD^2}} \qquad (3.15)
\end{aligned}$$

Example:

$$V = V_s \sin(\omega t)$$

$$\begin{aligned}
i(t) &= \text{square wave}\begin{cases} i_n = \frac{4}{\pi n}\left(\frac{i_{pk}}{2}\right) \\ i_0 = i_{ave} = \frac{1}{2} i_{pk} \end{cases} \\
THD &= 121\% \\
k_d &= \frac{\frac{i_{pk}}{2}\cdot\frac{4}{\pi}\cdot\frac{1}{\sqrt{2}}}{\frac{i_{pk}}{\sqrt{2}}} \\
&= \frac{2}{\pi} \\
P.F. &= 0.63 \qquad (3.16)
\end{aligned}$$

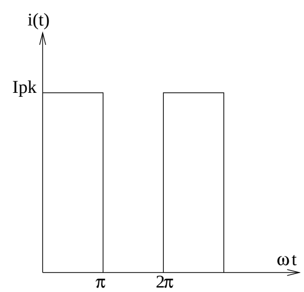

Figure 3.4: Example

(Passive) Power Factor Compensation (KSV: Section 3.4.1)

Lets focus on the displacement factor component of power factor. For simplicity, lets assume a linear load (e.g. R-L) so that voltages and currents are sinusoidal.

For sinusoidal V and i:

$$P.F. = \frac{< P >}{V_{RMS} i_{RMS}} = \cos\phi \qquad (3.17)$$

ϕ is the power factor angle:

- Leading $\phi < 0$ Capacitive
- Lagging $\phi > 0$ Inductive

Real power:

$$P = V_{RMS} I_{RMS} \cos\phi \tag{3.18}$$

Define reactive power as:

$$Q \doteq V_{RMS} I_{RMS} \sin\phi \tag{3.19}$$

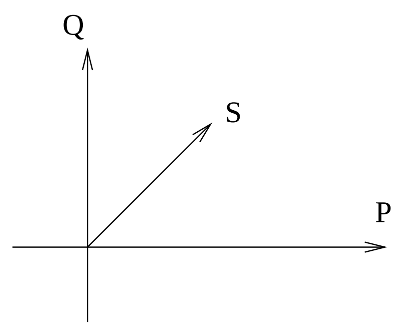

Figure 3.5: Reactive Power

In vector form $\vec{S} = P + jQ$. In phaser form $\vec{V}, \vec{i} \rightarrow \vec{S} = < VI^* >$

		units
Apparent Power	$S = \parallel \vec{S} \parallel = V_{RMS} I_{RMS}$	VA
Average Power	$Re\{S\} = P = V_{RMS} I_{RMS} \cos\phi$	W
Reactive Power	$Im\{S\} = Q = V_{RMS} I_{RMS} \sin\phi$	VAR

We can use these results to help adjust the displacement factor of a system. (make $Q_{net} \rightarrow 0$).

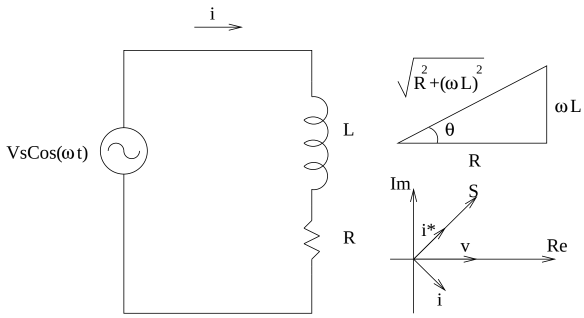

Figure 3.6: R-L Load

Suppose we have an R-L load (e.g. an induction machine):

$$
\begin{aligned}
i(t) &= \frac{V_s}{\sqrt{\omega^2 L^2 + R^2}} \cos(\omega t - \arctan(\frac{\omega L}{R})) \\
\text{since } S &\doteq V I^* \\
\text{voltage-current phase } \phi &= \arctan(\frac{\omega L}{R}) \\
P.F. &= \cos(\arctan(\frac{\omega L}{R})) \\
&= \frac{R}{\sqrt{R^2 + \omega^2 L^2}} < 1
\end{aligned}
\tag{3.20}
$$

We can add some additional reactive load to balance out and give net unity power factor.

$$
\begin{aligned}
S &= V_{RMS} I_{RMS} \\
&= \frac{V_s^2}{2\sqrt{\omega^2 L^2 + R^2}} \\
P &= S \cos\phi \\
&= V_{RMS} I_{RMS} \cos\phi
\end{aligned}
\tag{3.21}
$$

$$
\begin{aligned}
&= \frac{V_s^2 R}{2(\omega^2 L^2 + R^2)} && (3.22)\\
jQ &= jS\sin\phi \\
&= jV_{RMS} I_{RMS}\sin\phi \\
&= j\frac{\omega L V_s^2}{2(\omega^2 L^2 + R^2)} && (3.23)
\end{aligned}
$$

So we have real and reactive power.

Suppose we add a capacitor in parallel:

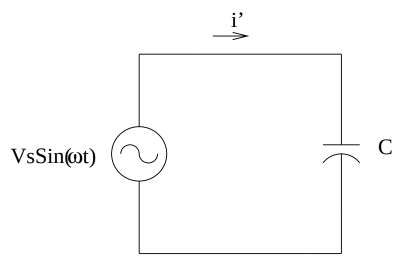

Figure 3.7: Capacitor

$$
\begin{aligned}
Z_c &= \frac{1}{j\omega C} \\
&= \frac{1}{\omega C} e^{-j\frac{\pi}{2}} \\
\frac{1}{Z_c} &= \omega C e^{j\frac{\pi}{2}} && (3.24)\\
V_{phase} - i_{phase} &= -90^\circ \\
i' &= V_s \omega C \sin(\omega t + \frac{\pi}{2}) && (3.25)\\
S' &= V_{RMS} I_{RMS} \\
&= \frac{1}{2} V_s^2 \omega C && (3.26)\\
P' &= 0 && (3.27)
\end{aligned}
$$

$$Q' = -j\frac{1}{2}V_s^2\omega C \tag{3.28}$$

So by placing the capacitor in parallel:

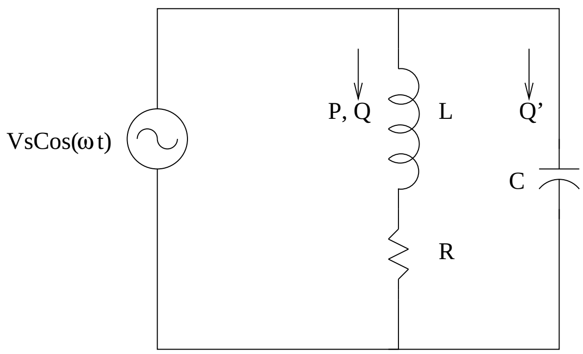

Figure 3.8: Parallel Capacitor

$$\begin{aligned}
S &= P + jQ + jQ' \\
\text{make } jQ \text{ and } jQ' \text{ cancel: } Q + Q' &= 0 \\
j\frac{\omega L V_s^2}{2(\omega^2 L^2 + R^2)} - j\frac{1}{2}V_s^2\omega C &= 0 \\
C &= \frac{L}{\omega^2 L^2 + R^2} \qquad (3.29)
\end{aligned}$$

Example:

$$\begin{aligned}
\omega &= 377 RAD/sec\ (\omega HZ) \\
R &= 1\Omega \\
L &= 2.7mH \\
\Rightarrow C &= 1.32mF
\end{aligned}$$

If we know our load, we can add reactive elements to compensate so that no displacement factor reduction of line utilization occurs. Real, reactive power definitions are useful to help us do this. This does not help with distortion factor.