

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 3
Fall Term 2006

6.630 Electromagnetic Theory

Issued: Week 3
Due: Week 4

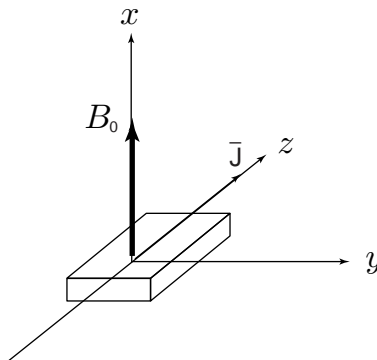
Reading assignment: Section 1.4-1.5, 1.9 ; J. A. Kong, “*Electromagnetic Wave Theory*,” EMW, 2005.

Problem P3.1

The Earth receives over all frequency bands about 1.5 kW/m^2 of power from the Sun.

- The Earth-Sun distance is $150 \times 10^9 \text{ m}$. How long does it take the sunlight to reach the Earth?
- The Earth radius is 6400 km. What is the total power received by the Earth?
- Assume the Sun’s mass is $2 \times 10^{30} \text{ kg}$ which converts to radiated energy according to mc^2 at 1 percent efficiency. How long can the Sun radiate at the present level?
- The Sun radiates $10^{-20} \text{ W m}^{-2} \text{ Hz}^{-1}$ at 3 GHz. Assuming constant power level over 1 GHz bandwidth, what is the Poynting power density and the corresponding electric field amplitude?

Problem P3.2



Consider a slab of semiconductor with positive charge carriers of density N such that there is a uniform current density of $\vec{J} = \hat{z}Nqv$ flowing in the $+\hat{z}$ direction as shown in above figure. Calculate the force density \vec{F} acting on the charges if a static magnetic field $\vec{B} = \hat{x}B_0$ is applied. This force will set up an accumulation of charges in the \hat{y} direction. What is the Hall voltage generated by the accumulation of charges?

Problem P3.3

Consider a current density J flowing through a resistor of conductivity σ , length l , and circular cross-section with radius a . The electric field is $E = J/\sigma$, and the magnetic field $H = aJ/2$. Show that the total Poynting power flow into the resistor is $\pi a^2 l J^2 / \sigma$.

Problem P3.4

Consider an electromagnetic wave propagating in free space with

$$\vec{E} = \hat{x}E_0 \cos(kz - \omega t)$$

Find the following terms:

- Magnetic field strength \vec{H}
- Poynting vector \vec{S} and its time-average $\langle \vec{S} \rangle$

- (c) Electric energy density W_e and its time-average $\langle W_e \rangle$
- (d) Magnetic energy density W_m and its time-average $\langle W_m \rangle$

Problem P3.5

Sun navigation was first observed in 1911. It was found that some species of ants, horseshoe crabs, honeybees, etc., are sensitive to polarized light. These creatures can navigate as long as there is a small patch of blue sky. The sky polarization depends upon the angle ϕ between the sun's rays to a particular point in the sky and an observer's line of sight to the same point. The sunlight, which is unpolarized, or randomly polarized, excites air molecules which behave like small dipole antennas when irradiated by the incident electric fields of the sunlight. The scattered electric field \bar{E}_s for each excited dipole antenna is linearly polarized in planes perpendicular to the sunlight path; and looking along the sun ray path the scattered wave is unpolarized, or randomly polarized.

At sunset, if an ant looks directly at the sun ($\phi = 0$), what is the polarization? What is the polarization if the ant looks at the zenith ($\phi = 90^\circ$) perpendicular to the sun ray path? Show that the sky light appears to be partially linearly polarized when it looks at other parts of the sky [*Scientific American*, July 1955].