# Lecture 28 - The "Long" Metal-Oxide-Semiconductor Field-Effect Transistor (cont.) 

April 18, 2007

## Contents:

## 1. Second-order and non-ideal effects

## Reading assignment:

del Alamo, Ch. 9, §9.7

## Key questions

- The potential of the inversion layer increases along the channel. This should change the local threshold voltage. Does this affect the I-V characteristics of the MOSFET?
- What happens to MOSFET I-V characteristics if we apply a bias to the body with respect to the source?


## 1. Second-order and non-ideal effects in MOSFETs

Introduce four significant refinements to model:

- Body effect (impact of $y$-dependence of $V_{T}$ )
- Back bias (impact of $V_{B S}$ )
- Channel length modulation (impact of $V_{D S}>V_{D S s a t}$ )
- Subthreshold regime (channel conduction for $V_{G S}<V_{T}$ )


## Body effect

In a MOSFET biased in linear or saturation regimes, channel voltage $V(y)$ depends on position:
$\Rightarrow$ voltage difference between channel and body $V(y)$
$\Rightarrow V_{T}(y)$ (increases along $y$ )

no body effect


with body effect


Dependence of $V_{T}(y)$ further debiases transistor:
$\Rightarrow I_{D}$ lower than ideal
$\Rightarrow V_{D S \text { sat }}$ lower than ideal

Voltage dependence of $V_{T}$ :

$$
V_{T}(V)=V_{T o}+\gamma\left(\sqrt{\phi_{s t h}+V}-\sqrt{\phi_{s t h}}\right)
$$

$V_{T o}$ is $V_{T}$ for $V_{S B}=0$.
Charge control relation becomes:
$Q_{i}=-C_{o x}\left(V_{G S}-V-V_{T}\right)=-C_{o x}\left[V_{G S}-V-V_{T o}-\gamma\left(\sqrt{\phi_{s t h}+V}-\sqrt{\phi_{s t h}}\right)\right]$

Insert into current equation:
$I_{e}=W \mu_{e} Q_{i} \frac{d V}{d y}=-W \mu_{e} C_{o x}\left[V_{G S}-V-V_{T o}-\gamma\left(\sqrt{\phi_{s t h}+V}-\sqrt{\phi_{s t h}}\right)\right] \frac{d V}{d y}$

Integrate from $y=0$ to $y=L \Rightarrow$ MOSFET current in linear regime:

$$
I_{D}=\frac{W}{L} \mu_{e} C_{o x}\left\{\left(V_{G S}-V_{T o}+\gamma \sqrt{\phi_{s t h}}-\frac{1}{2} V_{D S}\right) V_{D S}-\frac{2}{3} \gamma\left[\left(\phi_{s t h}+V_{D S}\right)^{3 / 2}-\left(\phi_{s t h}\right)^{3 / 2}\right]\right\}
$$

Note new terms multiplied by $\gamma \Rightarrow$ if $\gamma \rightarrow 0$, body effect $\rightarrow 0$.

To get $V_{D S s a t}$, look at $Q_{i}$ at $y=L$ :
$Q_{i}(y=L)=-C_{o x}\left[V_{G S}-V_{D S s a t}-V_{T o}-\gamma\left(\sqrt{\phi_{s t h}+V_{D S s a t}}-\sqrt{\phi_{s t h}}\right)\right]=0$

Solve for $V_{D S s a t}$ :

$$
V_{D S s a t}=V_{G S}-V_{T o}+\gamma \sqrt{\phi_{s t h}}-\frac{\gamma^{2}}{2}\left[\sqrt{1+\frac{4}{\gamma^{2}}\left(V_{G S}-V_{F B}\right)}-1\right]
$$

MOSFET saturated current: plug $V_{D s s a t}$ into current equation in linear regime:

$$
\begin{aligned}
I_{D s a t}= & \frac{W}{L} \mu_{e} C_{o x}\left\{\left(V_{G S}-V_{T o}+\gamma \sqrt{\phi_{s t h}}-\frac{1}{2} V_{D S s a t}\right) V_{D S s a t}\right. \\
& \left.-\frac{2}{3} \gamma\left[\left(\phi_{s t h}+V_{D S s a t}\right)^{3 / 2}-\left(\phi_{s t h}\right)^{3 / 2}\right]\right\}
\end{aligned}
$$



Three noticeable features:

- for all values of $V_{G S}$ and $V_{D S}$, body effect reduces $I_{D}$
- for given $V_{G S}$, body effect reduces $V_{D S s a t}$
- body effect goes away as transistor is turned off


## Key observations for model simplification:

- $V_{D S s s a t}$ dependence on $V_{G S}$ remains roughly linear:

- $I_{D s a t}$ dependence on $V_{G S}$ remains roughly quadratic:


Linearize dependence of $V_{T}$ on $V\left(V \ll \phi_{s t h}\right)$ :

$$
V_{T}(V)=V_{T o}+\gamma\left(\sqrt{\phi_{s t h}+V}-\sqrt{\phi_{s t h}}\right) \simeq V_{T o}+\frac{\gamma}{2 \sqrt{\phi_{s t h}}} V
$$

Solve again differential equation to get MOSFET current in linear regime:

$$
I_{D} \simeq \frac{W}{L} \mu_{e} C_{o x}\left(V_{G S}-V_{T o}-\frac{m}{2} V_{D S}\right) V_{D S}
$$

with:

$$
m=1+\frac{\gamma}{2 \sqrt{\phi_{s t h}}}>1
$$

$V_{D S s a t}$ becomes:

$$
V_{D S s a t} \simeq \frac{1}{m}\left(V_{G S}-V_{T o}\right)
$$

Current in saturation regime:

$$
I_{D s a t} \simeq \frac{W}{2 m L} \mu_{e} C_{o x}\left(V_{G S}-V_{T o}\right)^{2}
$$



Cite as: Jesús del Alamo, course materials for 6.720 Integrated Microelectronic Devices, Spring 2007.
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$m$ is body-effect coefficient $(m>1)$ :

$$
m=1+\frac{\gamma}{2 \sqrt{\phi_{s t h}}}
$$

$m$ has same dependences as $\gamma$ :

- $x_{o x} \downarrow \Rightarrow \gamma \downarrow \Rightarrow m \downarrow$ (less severe body effect)
- $N_{A} \uparrow \Rightarrow \gamma \uparrow \Rightarrow m \uparrow$ (more severe body effect)
$m$ and $\gamma$ represent relative electrostatic influence of gate and body on inversion layer; if $\gamma=0 \rightarrow m=1$ (negligible impact of body).

In circuit CAD, $m$ used as fitting parameter. Typically $m \sim 1.1-1.4$.

## Back bias

If bias applied to body with respect to source $\left(V_{S B}>0\right)$ :

$$
\begin{aligned}
& \Rightarrow V_{T} \text { shifts positive } \\
& \Rightarrow \text { for constant } V_{G S} \text { and } V_{D S}, I_{D} \text { reduced }
\end{aligned}
$$

Model in absence of body effect $\Rightarrow$ just replace $V_{T}$ in first order model by:

$$
V_{T}\left(V_{S B}\right)=V_{T o}+\gamma\left(\sqrt{\phi_{s t h}+V_{S B}}-\sqrt{\phi_{s t h}}\right)
$$




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## I-V characteristics of n-channel MOSFET ( $L=1.5 \mu m$ )

$\square$ Output characteristics $\left(V_{G S}=0-3 V, \Delta V_{G S}=0.5 V\right)$ :


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$\square$ Transfer characteristics $\left(V_{D S}=4 V\right)$ :


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$\square$ Output characteristics vs. back bias $\left(V_{S B}=0,2 V\right)$ :



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$\square$ Transfer characteristics vs. back bias $\left(V_{D S}=4 V, V_{S B}=0-2 V\right.$, $\left.\Delta V_{S B}=0.5 \mathrm{~V}\right)$ :

$\square$ Backgate output characteristics ( $V_{S B}=0-3 V$ in $0.5 V$ increments, $V_{G S}=1.5 \mathrm{~V}$ ):


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## Key conclusions

- "Body effect" arises from spatial dependence of $V_{T}$ : local gate overdrive reduced.
- Main consequences of body effect:
- $I_{D}$ lower than ideal,
- $V_{D S s a t}$ lower than ideal.
- Simple formulation of body effect is fairly accurate:

$$
I_{D s a t} \simeq \frac{W}{2 m L} \mu_{e} C_{o x}\left(V_{G S}-V_{T o}\right)^{2}
$$

with $m \geq 1$.

- $m$ captures relative electrostatic influence of gate and body (want $m \rightarrow 1)$.
- Application of back bias shifts $V_{T}$ positive and reduces $I_{D}$.

