Feedback

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***(with thanks to SDS)**

Outline

- > **Motivation for using feedback**
- > **The uses of (linear) feedback**
- > **Feedback on Nonlinear Systems**
  - **Quasi-static systems**
  - **Oscillators**

Why use feedback?

- For actuators, how do you know when you have actuated?
  - You can calibrate/calculate/etc., but what about drifts?
- Adding a sensor can tell you ***where you are***
- Combining the sensor + actuator with feedback can <u>keep</u> you where you are

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An optical attenuator that uses
- **MEMS *actuator***
- ***Senses* optical output**
- **Uses *feedback* to control attenuation**

Why use feedback?

- For sensors, feedback can be used to enhance sensor response
  - E.g., keep sensitivity constant
- Must add an *actuator* to do this

An accelerometer that uses
- MEMS tunneling *sensor*
- Electrostatic *actuation*
- Uses *feedback* to control tunneling current (and thus gap)

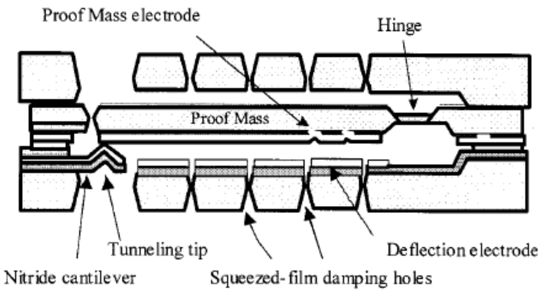


Figure 1 on p. 426 in: Liu, C.-H., and T. W. Kenny. "A High-precision, Wide-bandwidth, Micromachined Tunneling Accelerometer." *Journal of Microelectromechanical Systems* 10, no. 3 (September 2001): 425-433. © 2001 IEEE.

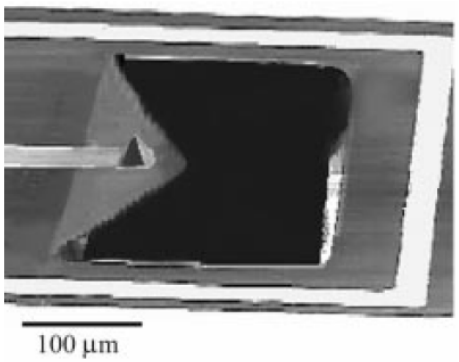


Figure 3a) on p. 426 in: Liu, C.-H., and T. W. Kenny. "A High-precision, Wide-bandwidth, Micromachined Tunneling Accelerometer." *Journal of Microelectromechanical Systems* 10, no. 3 (September 2001): 425-433. © 2001 IEEE.

Feedback in MEMS

- **Since MEMS is often concerned with making sensors for measurements or actuators to do something, feedback is integral to the subject**
- **Here we will examine some of the basic uses of feedback**
  - **Limit sensitivity to variations**
  - **Speed up system**
  - **Stabilize unstable systems**
- **At the end, we will look at feedback in nonlinear systems, which is useful for making oscillators**

Outline

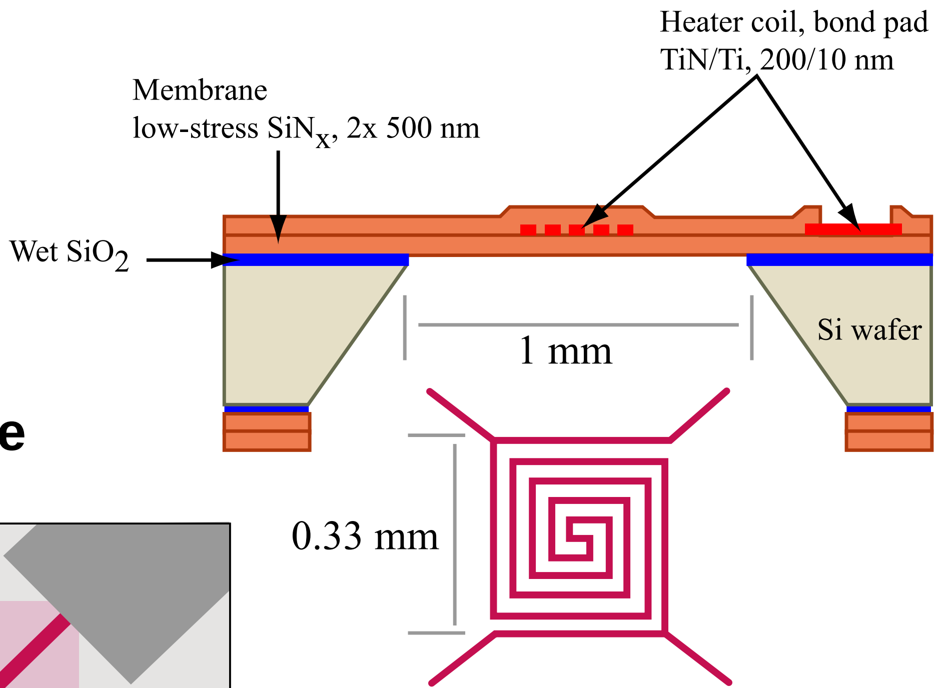
> **Motivation for using feedback**

> **The uses of (linear) feedback**

> **Feedback on Nonlinear Systems**
- **Quasi-static systems**
- **Oscillators**

Example: A MEMS hotplate

- Used for gas sensor
- Heat up active material, which reacts with gas and changes resistance
- Thermal MEMS is used because
  - Low power
  - Fast
  - Arrayable



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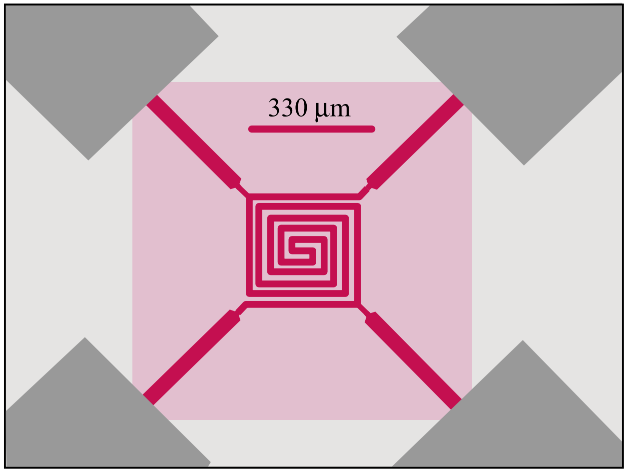

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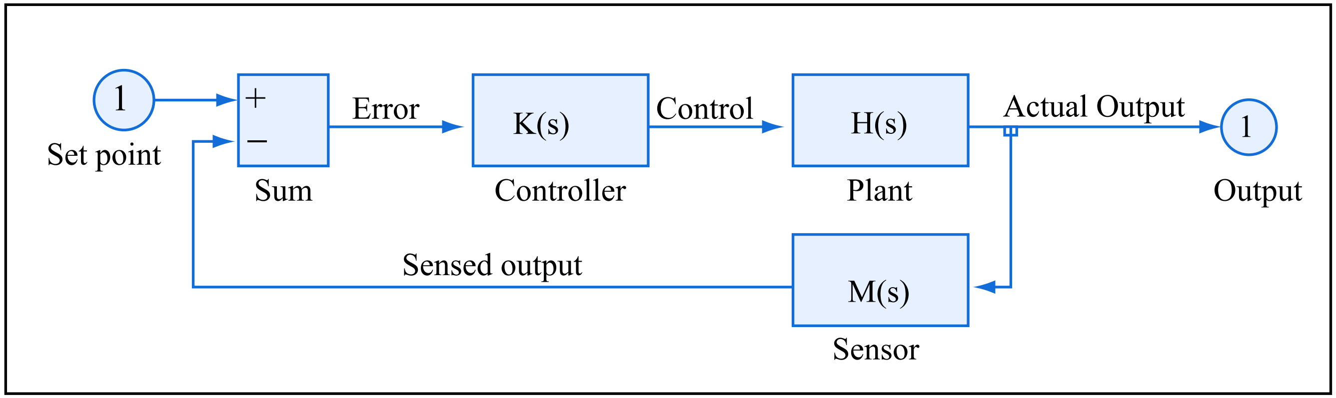
The Canonical Feedback System

- In controls, terminology refers to the plant, the controller, the state sensor, and the comparison point



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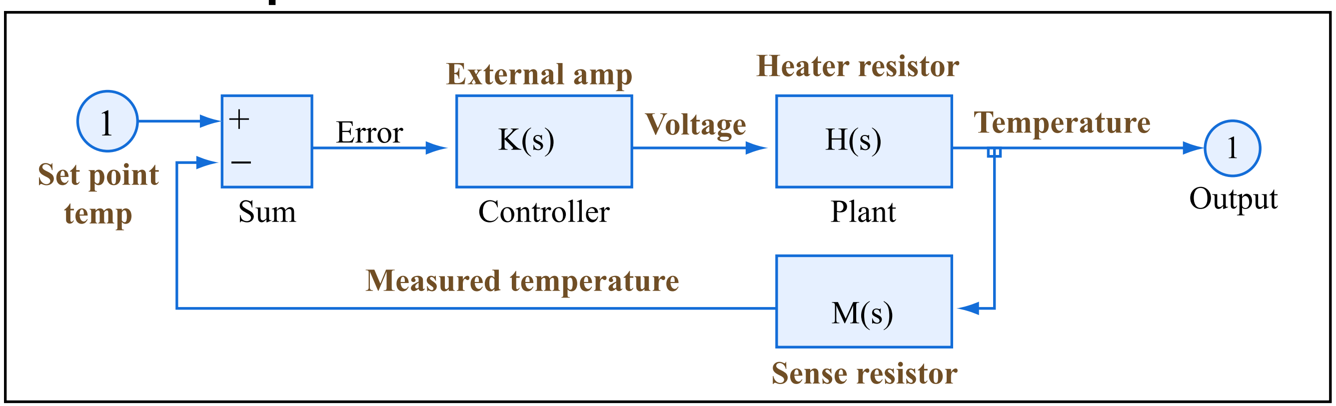
Adapted from Figure 15.1 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 397. ISBN: 9780792372462.

Example: micro hotplate



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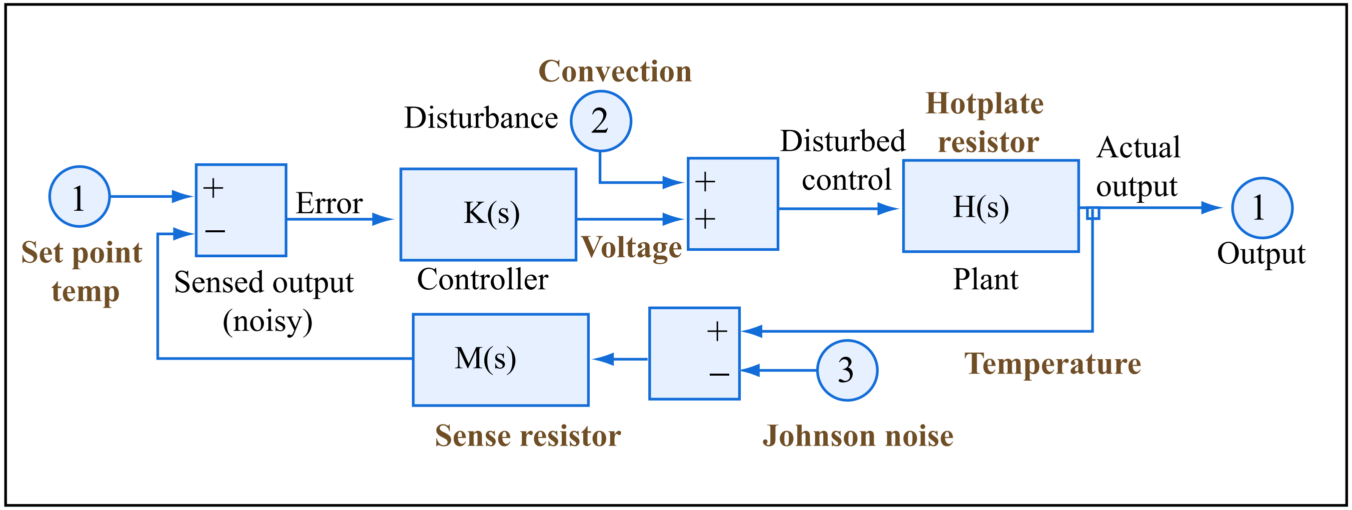
Adding Noise and Disturbances

- Noise corrupts the sensor output
- Disturbances modify the control input to the plant
- In some cases, what we want to measure is the disturbance (a feedback-controlled accelerometer)



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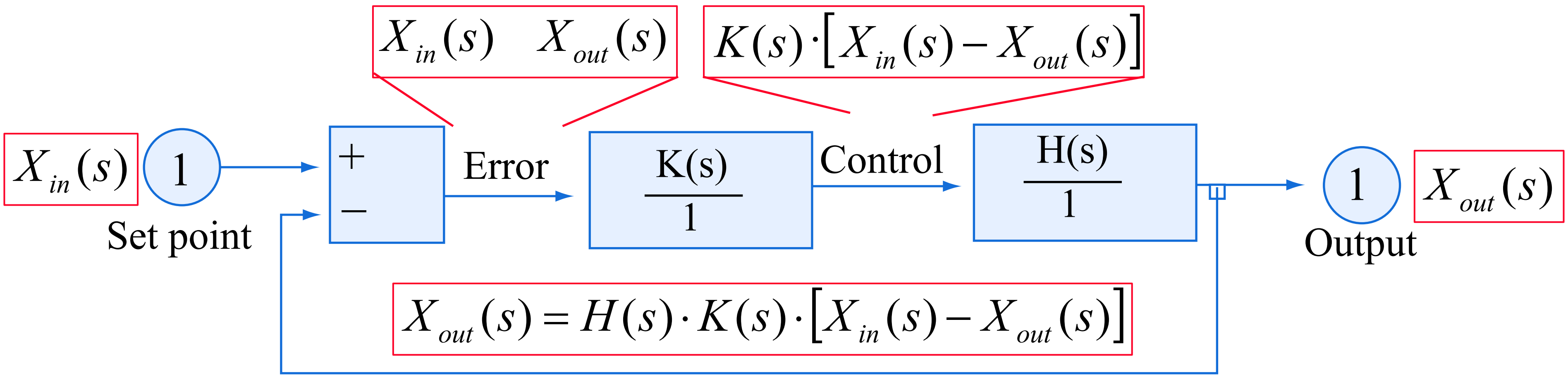
Adapted from Figure 15.2 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 398. ISBN: 9780792372462.

Linear Feedback: Black's Formula

- For a LTI system, we can use Laplace transforms to create an algebraic closed-loop transfer function
  - Assume sensor has (perfect) unity transfer function



$$\Downarrow$$

$$X_{out}(s)=\frac{H(s)\cdot K(s)}{1+H(s)\cdot K(s)}X_{in}(s)$$

Black's formula

Image by MIT OpenCourseWare.
Adapted from Figure 15.3 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 399. ISBN: 9780792372462.

$$\frac{X_{out}(s)}{X_{in}(s)}=\frac{forward\ gain}{1-loop\ gain}$$

Open-Loop Operation

> Control hot plate via calibration

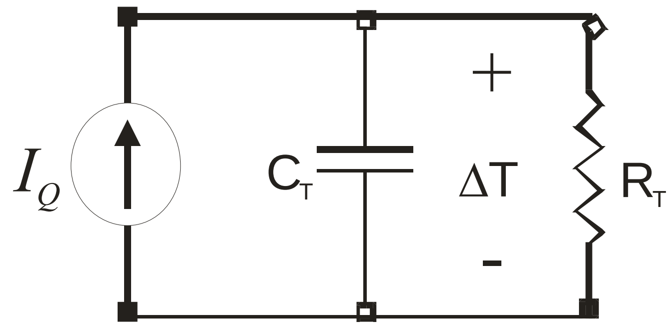
- Assume hotplate has 1st-order response with s_0~400 rad/s (f_0~65 Hz)
- Assume controller has no dynamics

$$H(s) = \frac{A_0 s_0}{s + s_0} \qquad K(s) = K_0$$

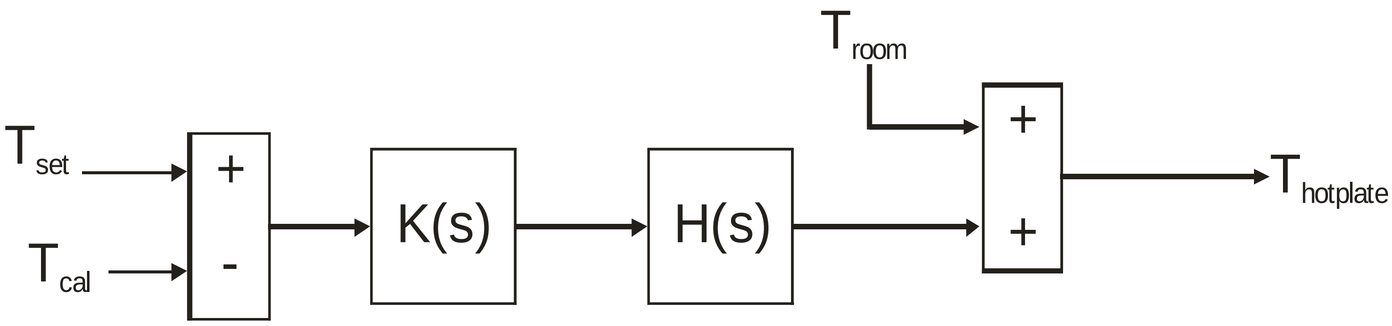
> Works great if there are no disturbances or drifts in system

> Any deviations cause steady-state error

Remember…



$$\frac{\Delta T}{I_Q} = \frac{R_T}{1 + R_T C_T s} = \frac{R_T \, {}^{1}\!/\!{}_{R_T C_T}}{{}^{1}\!/\!{}_{R_T C_T} + s}$$

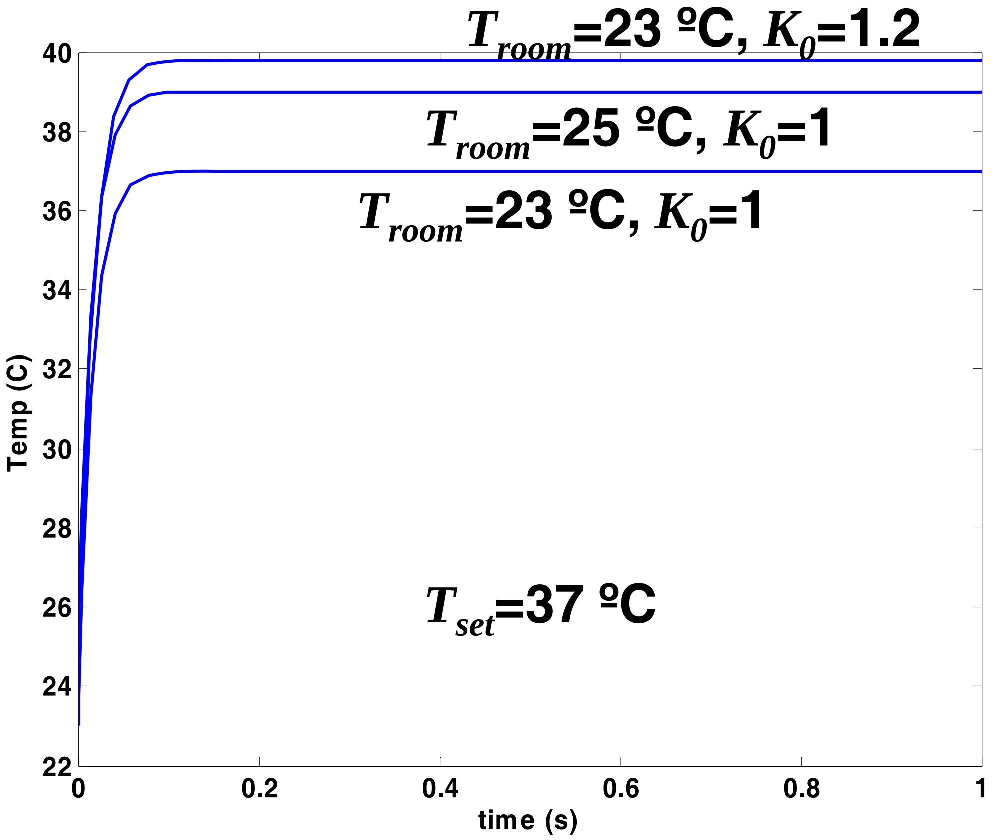


$$T_{hotplate} = T_{room} + H(s)K(s)(T_{set} - T_{cal})$$

Open-Loop Operation

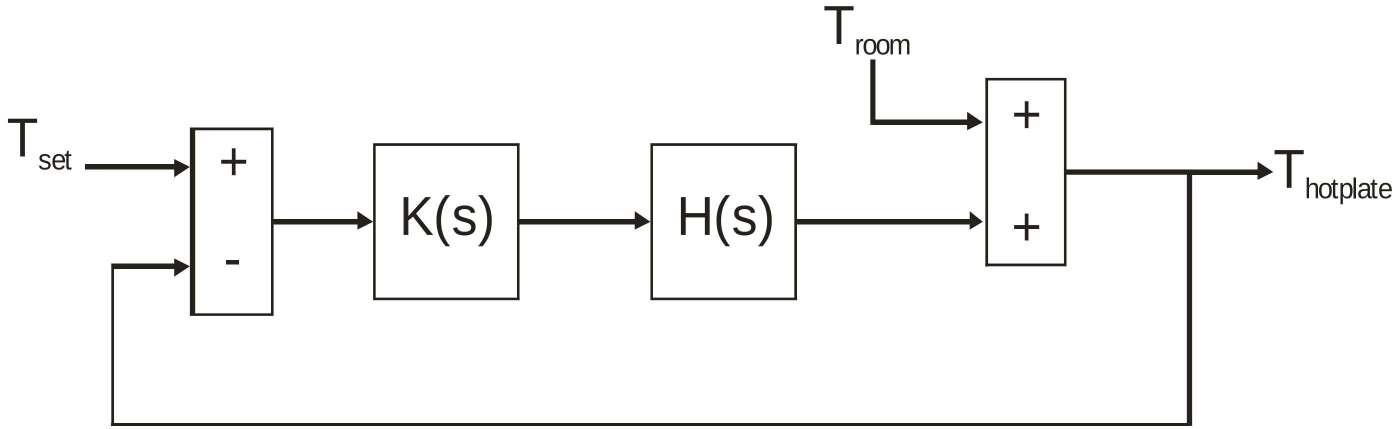
> **Response is sensitive to variations in controller, plant, and disturbances**

A_0=1
T_{cal}=23 ºC



Feedback use #1: limit sensitivity to variations

- **Add in term proportional to error**
- **This is called *proportional control***



$$T_{hotplate} = \frac{HK}{1+HK} T_{set} + \frac{1}{1+HK} T_{room}$$

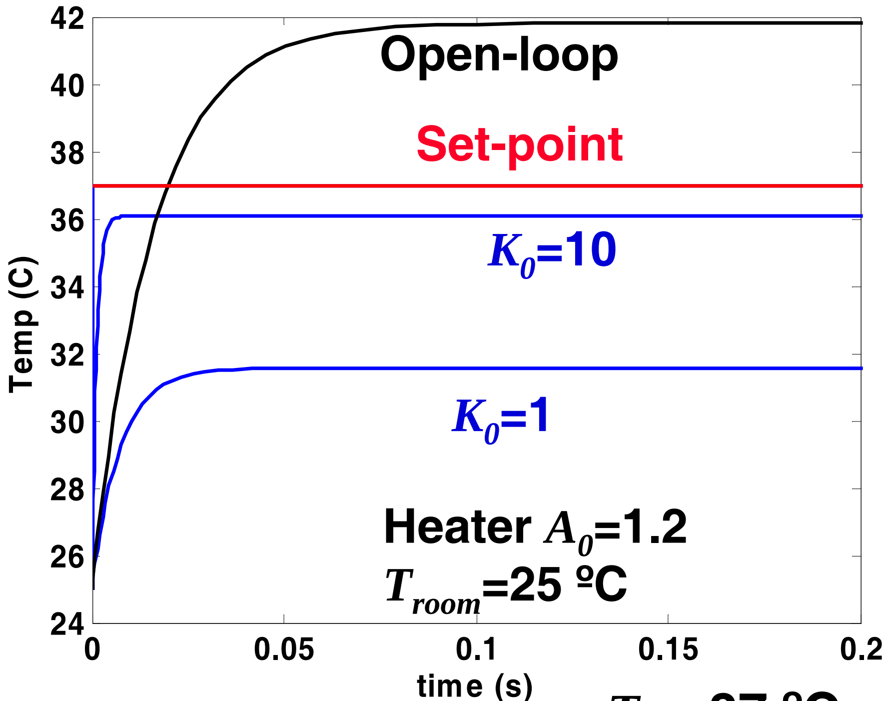
$$= \frac{\frac{A_0 s_0}{s+s_0} \cdot K_0}{1 + \frac{A_0 s_0}{s+s_0} \cdot K_0} T_{set} + \frac{1}{1 + \frac{A_0 s_0}{s+s_0} \cdot K_0} T_{room}$$

Closed-loop TF

$$T_{hotplate} = \frac{A_0 K_0 s_0}{s + s_0 \left(1 + A_0 K_0\right)} T_{set} + \frac{s + s_0}{s + s_0 \left(1 + A_0 K_0\right)} T_{room}$$

Close the loop

- Error → 0 as K_0 increases, despite
  - Variations in device (A_0)
  - Variations in plant (K_0)
  - Disturbances (T_{room})
- In limit of large K_0, system responds "perfectly"
  - Though DC error never goes exactly to zero



T_{set}=37 ºC

Steady-state (DC) Error

$$\varepsilon(s)\Big|_{s=0} = T_{set} - T_{hotplate} = \frac{1}{1+A_0K_0}T_{set} - \frac{1}{1+A_0K_0}T_{room}$$

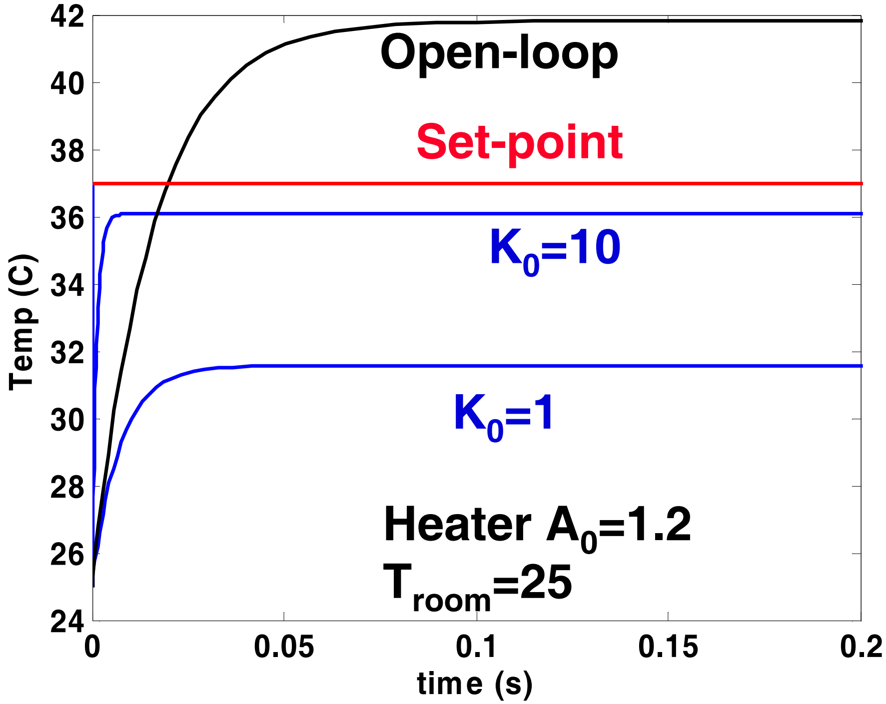
Feedback use #2: increase system bandwidth

- Settling time goes down
- Bandwidth goes up

$$H(s) = \frac{A_0 s_0}{s + s_0}$$

$$\frac{T_{hotplate}}{T_{set}} = H_{cl}(s) \approx \frac{A_0 K_0 s_0}{s + s_0\left(1 + A_0 K_0\right)}$$

$$\boxed{s_0 \rightarrow s_0\left(1 + A_0 K_0\right)}$$



Controlling a 2nd-order system

> Vibration sensor
  - Really just an z-axis accelerometer

> Use feedback to keep gap constant

> In this case, control signal measures vibration

> Mechanical "plant" is a SMD

$$H(s) = \frac{X_{out}(s)}{F(s)} = \left(\frac{1}{k}\right) \frac{1}{\hat{s}^2 + \frac{1}{Q}\hat{s} + 1}$$

$$\text{where: } \hat{s} = {s}/{\omega_0}, \omega_0 = \sqrt{{k}/{m}}, Q = {m\omega_0}/{b}$$

- **Set Q=1/2 (critically damped)**
- **Set k=1 for convenience**

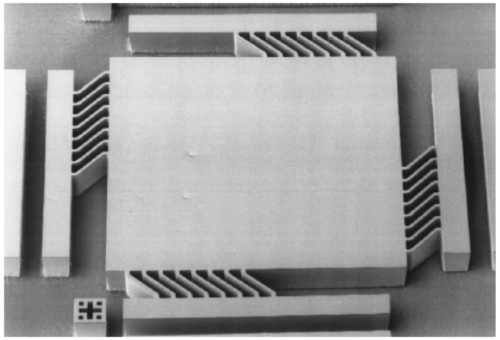


Figure 4 on p. 435 in: Bernstein, J., R. Miller, W. Kelley, and P. Ward. "Low-noise MEMS Vibration Sensor for Geophysical Applications." *Journal of Microelectromechanical Systems* 8, no. 4 (December 1999): 433-438.

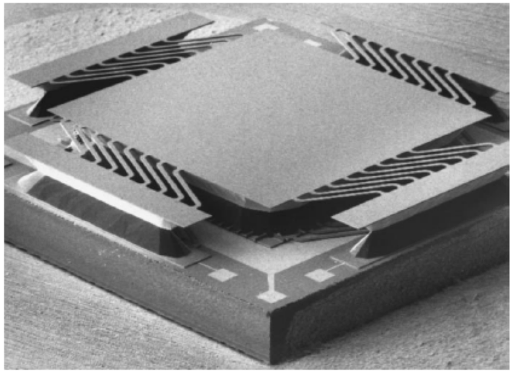


Figure 6 on p. 435 in: Bernstein, J., R. Miller, W. Kelley, and P. Ward. "Low-noise MEMS Vibration Sensor for Geophysical Applications." *Journal of Microelectromechanical Systems* 8, no. 4 (December 1999): 433-438.

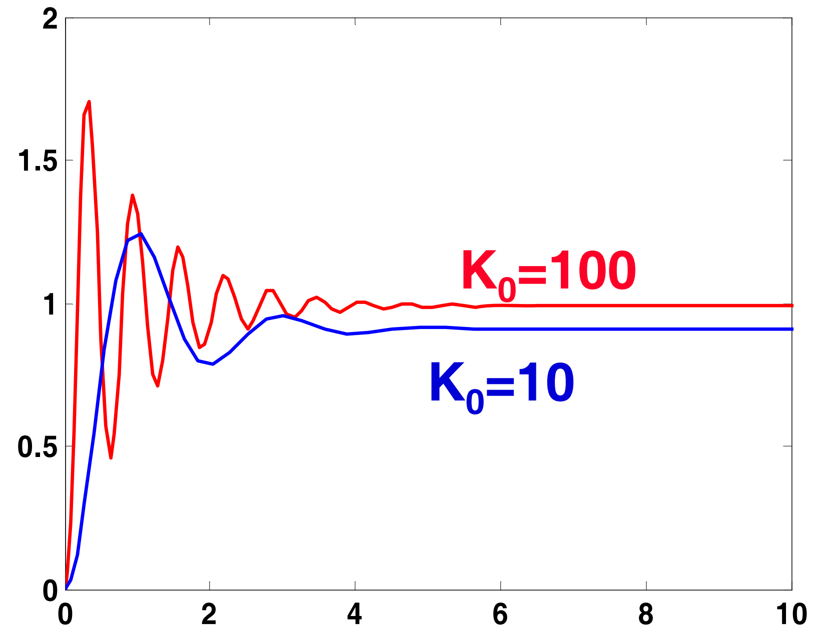
Proportional control of 2nd-order system

- Use ideal controller $K(s)=K_0$
- This gives us two overall poles:
  - Two from SMD $H(s)$
  - None from controller $K(s)$
- Some results are same as 1st-order system
  - Decreasing DC error as K_0 increases
  - System speeds up
- Some differences:
  - Q of closed-loop response increases with increasing DC gain
- This means that our critically damped system is now underdamped
  - This can be *bad* or *fatal* for our system

$$\frac{X_{out}(s)}{X_{in}(s)} = \frac{H(s)K(s)}{1+H(s)K(s)} = \frac{K_0}{\hat{s}^2 + \frac{1}{Q}\hat{s} + (K_0+1)}$$

$$\omega_{0,cl} = \sqrt{K_0+1}$$

$$Q_{cl} = Q\sqrt{K_0+1}$$



Control of complex systems

- Dynamics of closed-loop system are determined by H(s)K(s)
- Thus, behavior seen with 2nd-order SMD system will also occur with 1st-order thermal system coupled to 1st-order controller
- What happens when we add an additional pole?

Single-Pole Controller (Real amp)

- **Take SMD and control with 1st-order controller**
- **The system now has three poles**
- **When going to large K_0, the system goes unstable**
  - **This happens if one of the roots has real positive part**
- **Routh test can be used to find maximum gain**

$$K(s) = \frac{K_0}{1 + \hat{s}\hat{\tau}} \qquad \hat{\tau} = \omega_0 \tau$$

controller time constant

$$\Downarrow$$

$$\frac{X_{out}(s)}{X_{in}(s)} = \frac{K_0}{\hat{\tau}\hat{s}^3 + (1 + 2\hat{\tau})\hat{s}^2 + (2 + \hat{\tau})\hat{s} + K_0 + 1}$$

Routh test for third-order system:

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

All coefficients (a_n) must have same sign

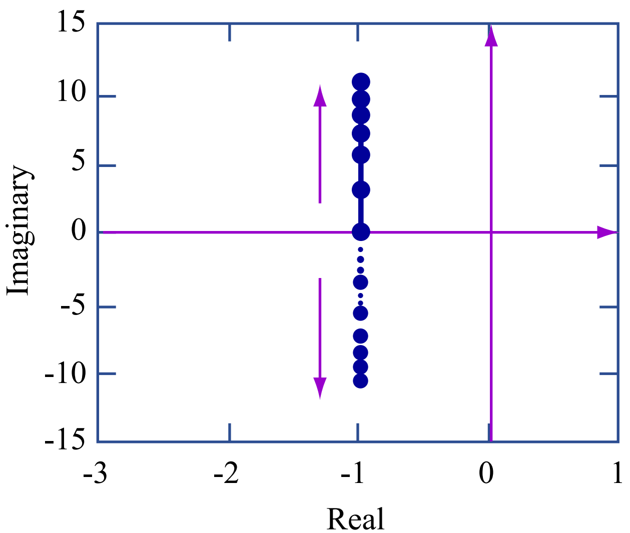
$$\text{AND} \quad a_2 a_1 > a_0 a_3$$

$$\Downarrow$$

$$K_0 < \frac{1}{Q}\left(\frac{1}{Q} + \hat{\tau} + \frac{1}{\hat{\tau}}\right)$$

2^{nd}-order vs. 3^{rd}-order systems

> **Stable system at all loop gains**



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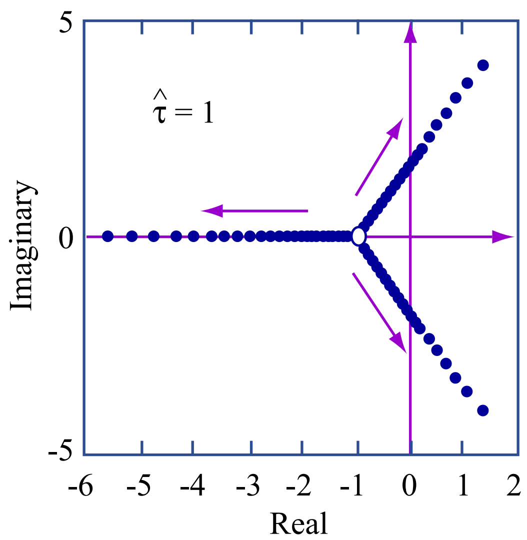
Boston, MA: Kluwer Academic Publishers, 2001, p. 402. ISBN: 9780792372462.

> **Unstable system at sufficiently high loop gain**



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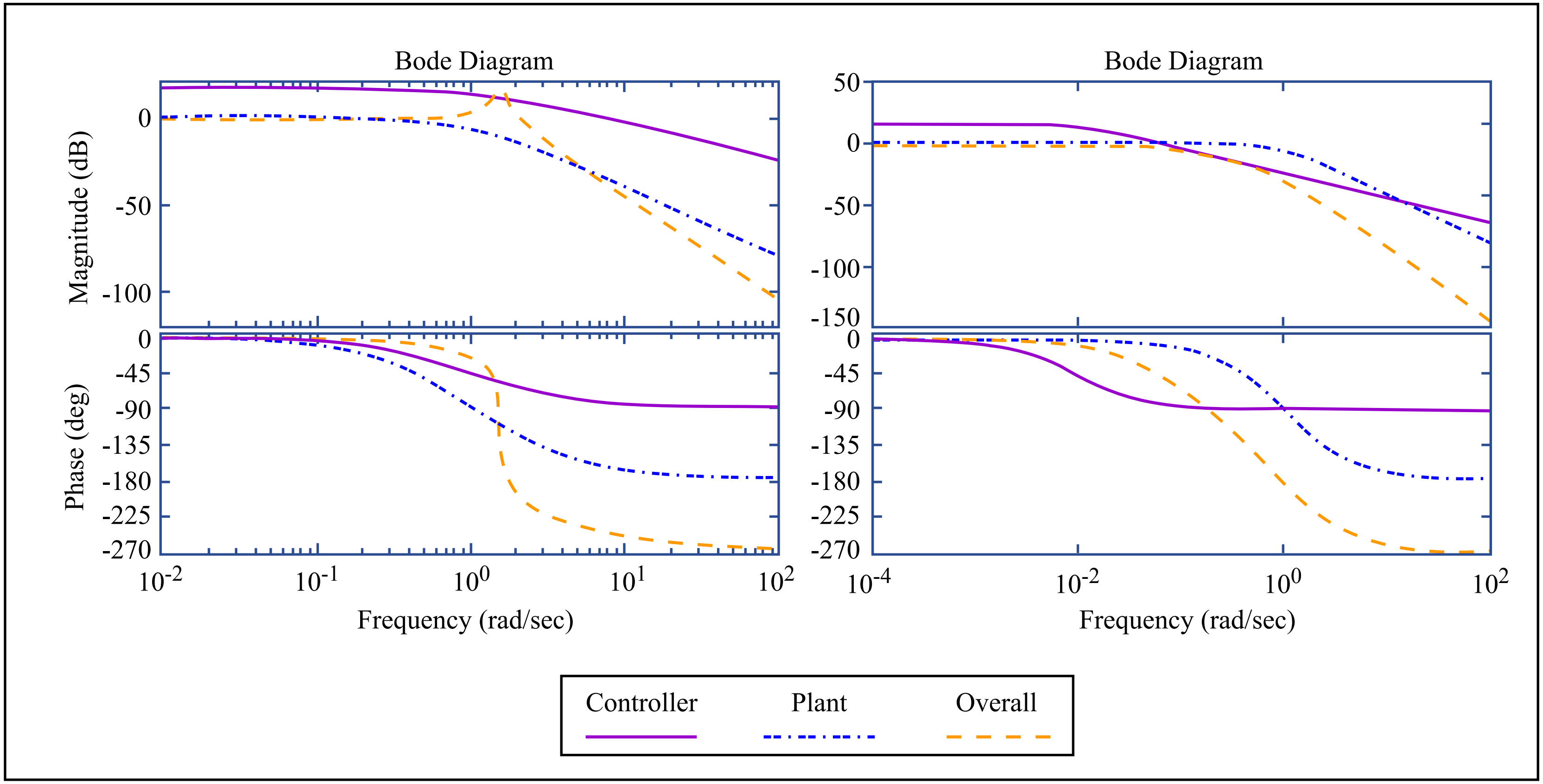
Boston, MA: Kluwer Academic Publishers, 2001, p. 403. ISBN: 9780792372462.

Effect of controller bandwidth

- Controller bandwidth < Plant bandwidth causes controller to dominate overall response

$Q=1/2$
$\omega_0=1$
$K_0=6$

$\tau=1$ $\tau=100$



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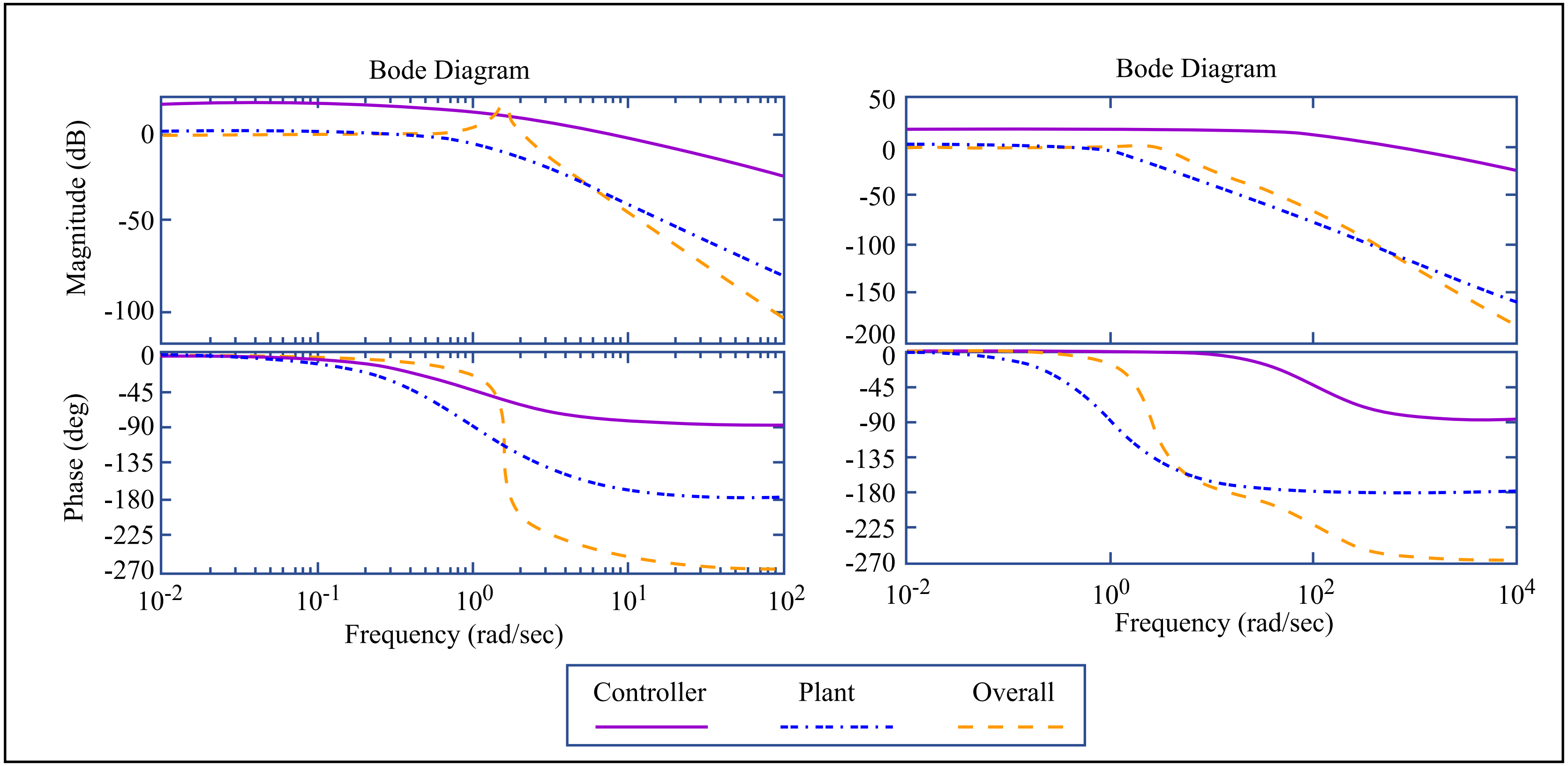
Effect of controller bandwidth

- Controller bandwidth > Plant bandwidth causes plant to dominate overall response

$Q=1/2$
$\omega_0=1$
$K_0=6$

$\tau=1$ $\tau=0.01$



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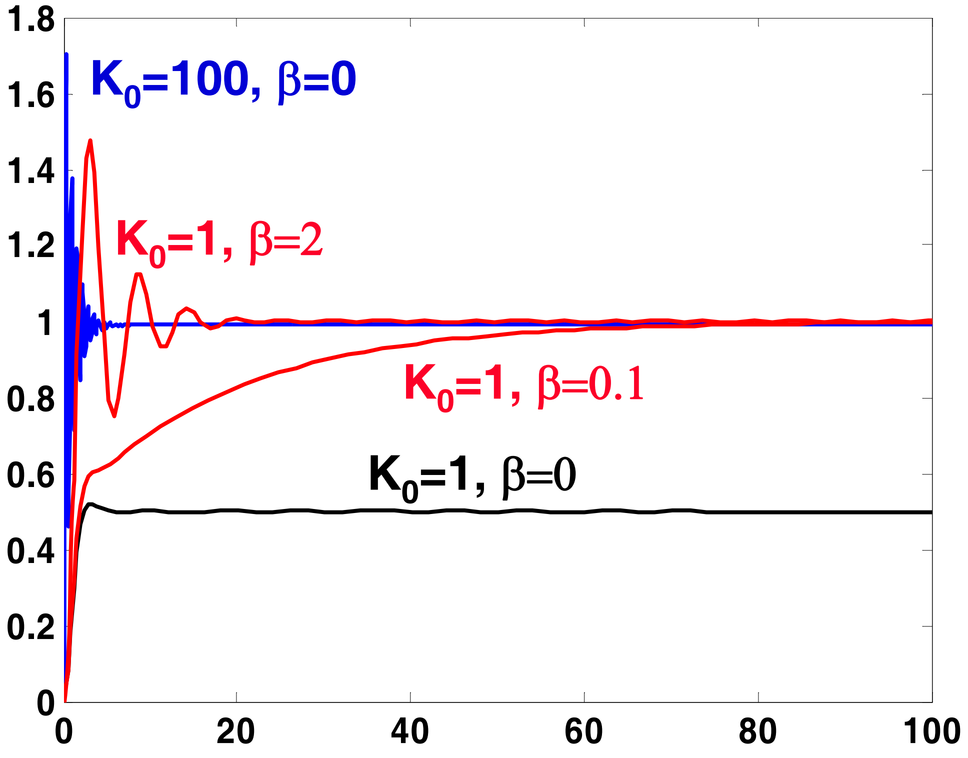
PI control

- Add pole at s=0
- This gives K(0) → ∞
  - And thus **no DC error**
- Benefits
  - As long as β≠0, will get perfect DC tracking, but it may take awhile
  - Completely insensitive to changes in plant at DC
- Drawbacks
  - Additional pole means possibility of ringing and instability

Proportional - Integral (PI) Control :

$$K(s) = K_0\left(1+\frac{\beta}{s}\right)$$

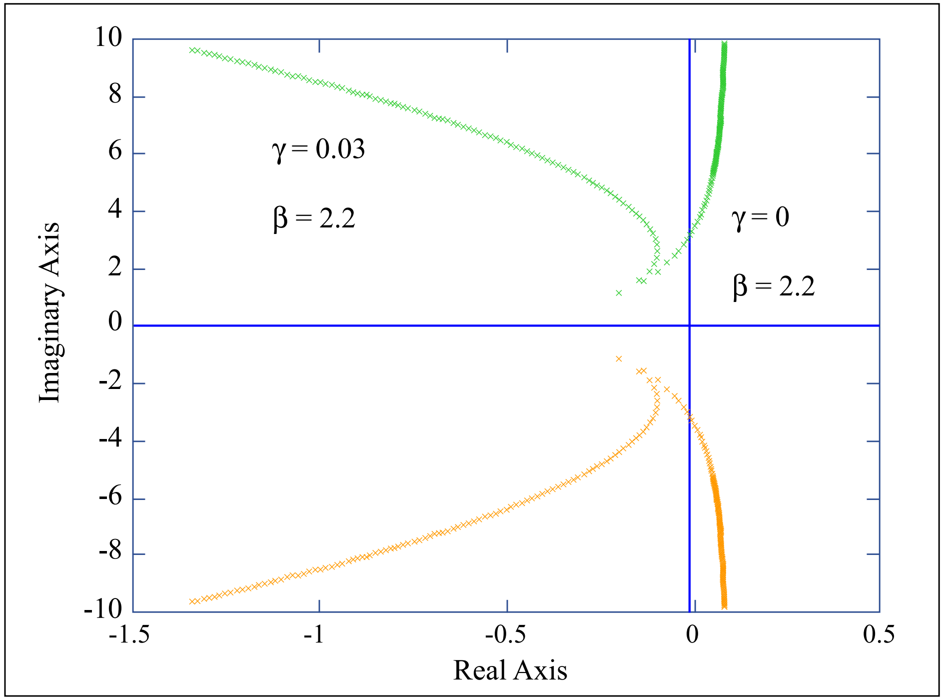


PID control

- Final generic term is to add in differential feedback
  - Anticipate future
- "Tame" ringing and instability due to integral and proportional control
- Methods exist to tune PID controllers

Proportional - Integral - Differential (PID) Control:

$$K(s) = K_0\left(1 + \frac{\beta}{s} + \gamma s\right)$$



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Stabilization of unstable systems

- Use of feedback #3: Stabilize an unstable system
- Stabilize electrostatic actuator beyond pull-in
- Most approaches use feedback to approximate charge control

Piyabongkarn (2005), IEEE Trans. Control Systems Tech.

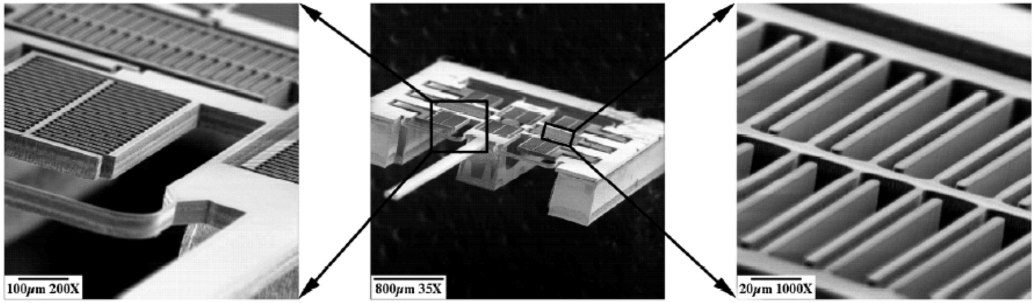


Figure 2 on p. 139 in: Piyabongkarn, D., Y. Sun, R. Rajamani, A. Sezen, and B. J. Nelson. "Travel Range Extension of a MEMS Electrostatic Microactuator." *IEEE Transactions on Control Systems Technology* 13, no. 1 (January 2005): 138-145 © 2005 IEEE.

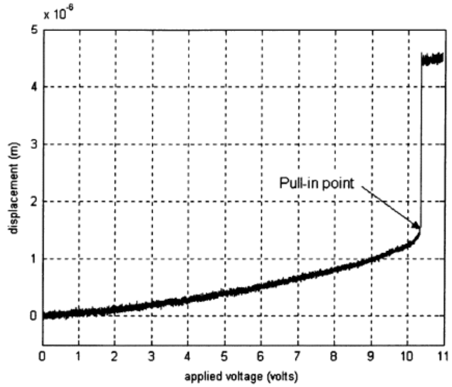


Figure 6 on p. 140 in: Piyabongkarn, D., Y. Sun, R. Rajamani, A. Sezen, and B. J. Nelson. "Travel Range Extension of a MEMS Electrostatic Microactuator." *IEEE Transactions on Control Systems Technology* 13, no. 1 (January 2005): 138-145. © 2005 IEEE.

no feedback

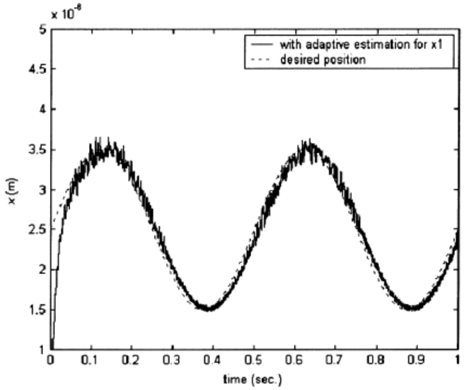


Figure 11 on p. 144 in: Piyabongkarn, D., Y. Sun, R. Rajamani, A. Sezen, and B. J. Nelson. "Travel Range Extension of a MEMS Electrostatic Microactuator." *IEEE Transactions on Control Systems Technology* 13, no. 1 (January 2005): 138-145. © 2005 IEEE.

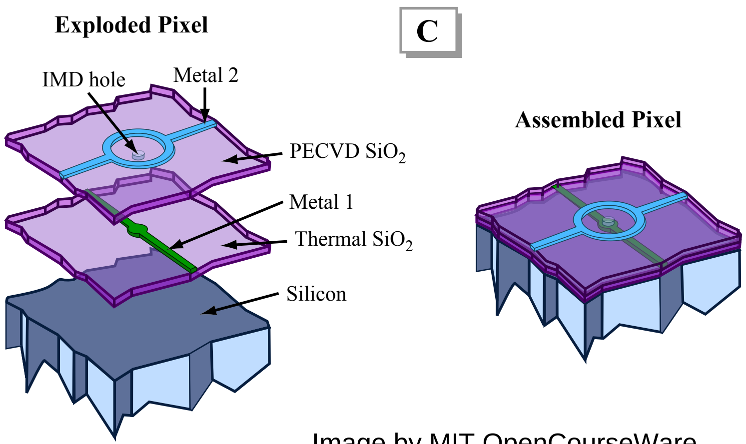
with feedback

Stabilization of unstable systems

- **However:**
  - **All potentially unstable modes must be both observable and controllable**
  - **Observable means that the sensor provides state information about the mode**
  - **Controllable means that the control inputs can modify the mode**
  - **If a mode has both attributes, it can be stabilized (at least in theory) with feedback**
- **Adding sensors to a system improves observability of modes**
- **Adding actuators improves controllability**
- **This can be generalized from unstable to unwanted…**

Control for MEMS

- Electrostatic traps for cells
- The goal is to trap single cells at each site
- System is currently run open loop
- Could we do better if we ran closed-loop?
- Need to sense: optical or electrical
- Need to actuate
  - This is hard…



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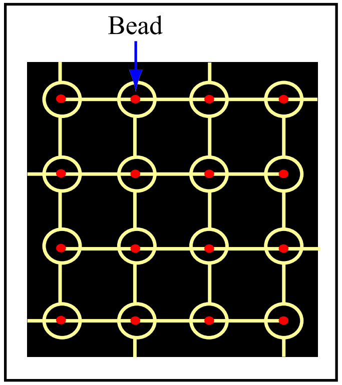

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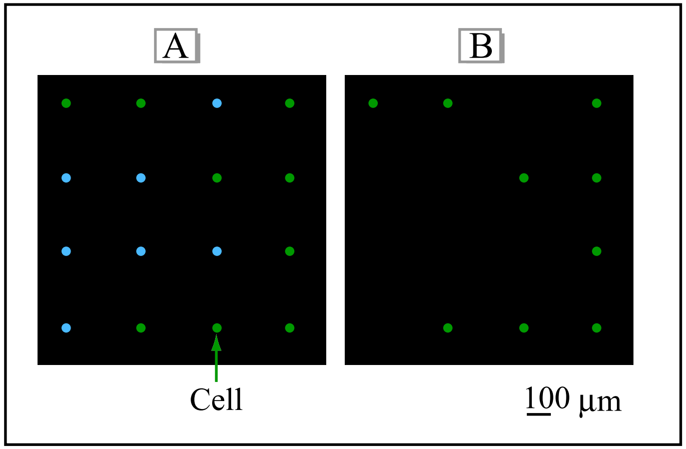

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Control for MEMS

- Many MEMS devices/systems are run open loop – why?
- Open loop
  - Does not need additional sensors or actuators
    - These increase fab complexity, chip size, cost, etc.
  - But is sensitive to perturbations
- Closed loop
  - Requires extra complexity
  - More stable performance
- If you don't need closed-loop control, don't use it

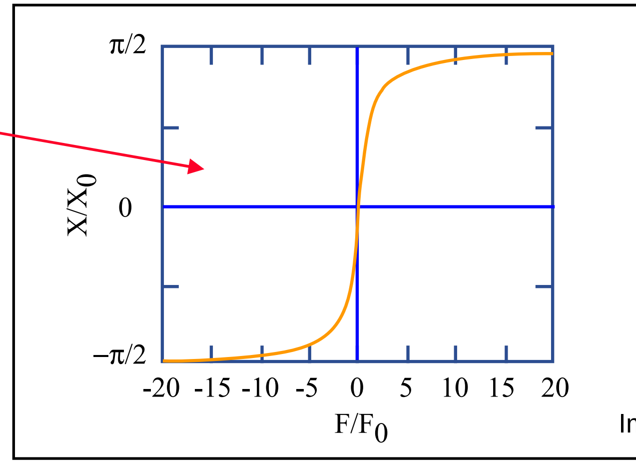
Outline

- Motivation for using feedback
- The uses of (linear) feedback
- Feedback of Nonlinear Systems
  - Quasi-static systems
  - Oscillators

Feedback in Nonlinear Systems

- Can no longer use nice algebraic forms
- However, the same idea still holds:
  - The controller pre-distorts the control signal so as to compensate for nonlinearities in the plant

$$X_{out} = X_0 \tan^{-1}\left[\frac{F}{F_0}\right]$$



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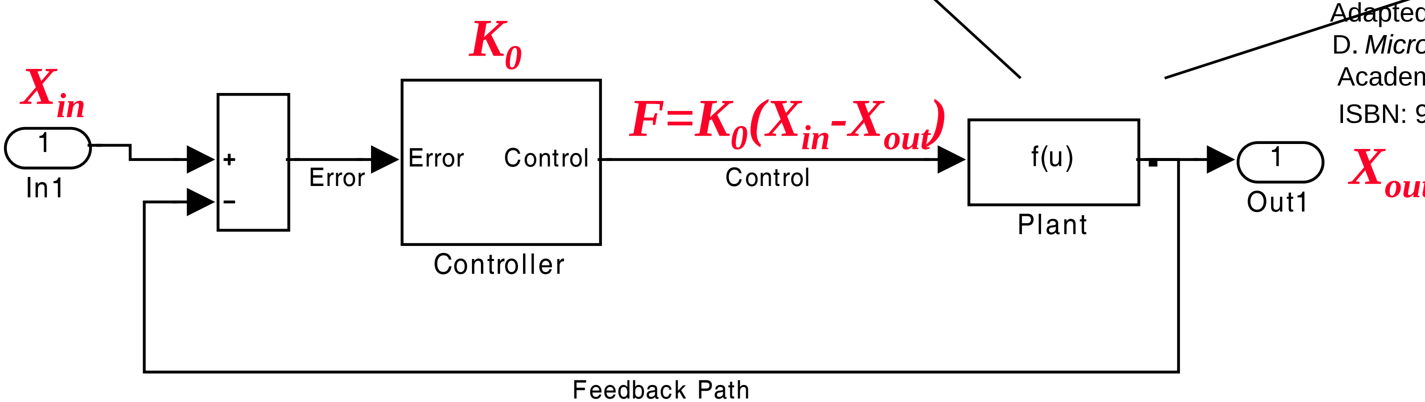


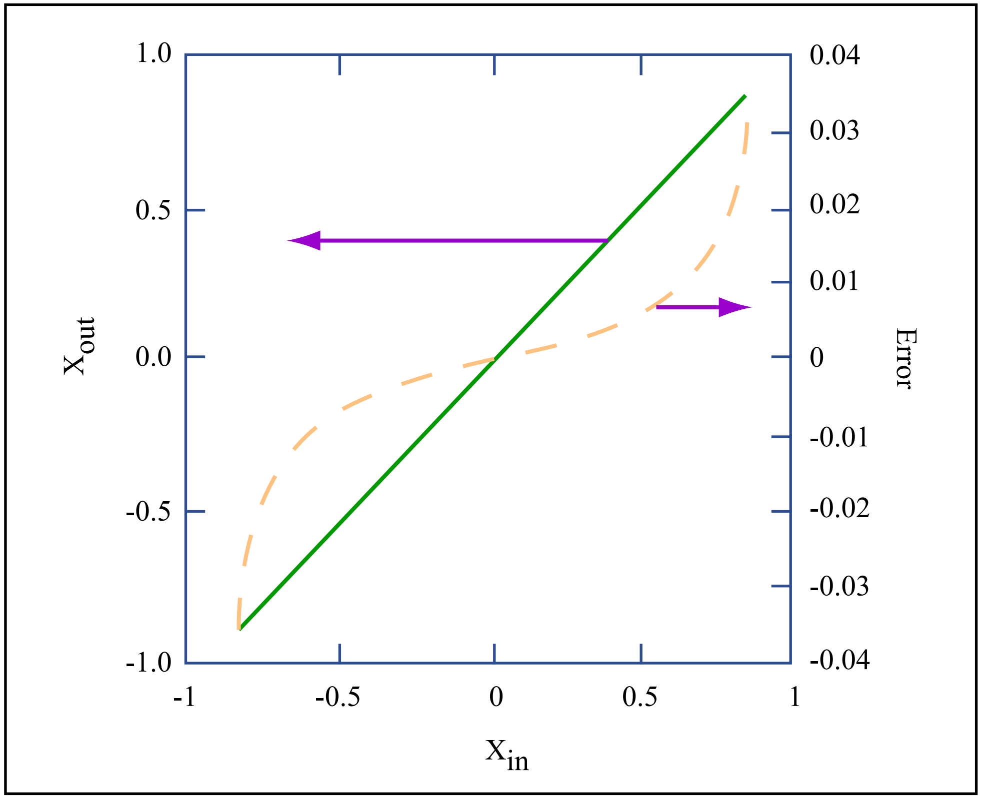
Feedback in Nonlinear Systems

- Controller "linearizes" in nonlinear system
- This also occurs in op-amps

$$X_{out} = X_0 \tan^{-1}\left[\frac{K_0\left(X_{in} - X_{out}\right)}{F_0}\right]$$

$$X_{in} - X_{out} = \frac{F_0}{K_0}\tan\left[\frac{X_{out}}{X_0}\right]$$

$$X_{in} - X_{out} \approx 0 \text{ for } K_0 >> F_0$$



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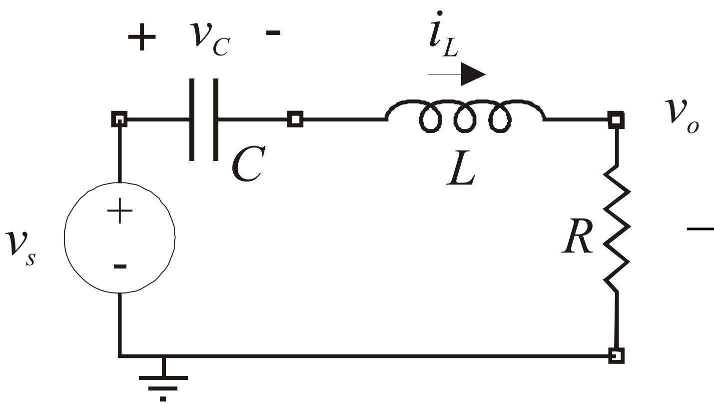
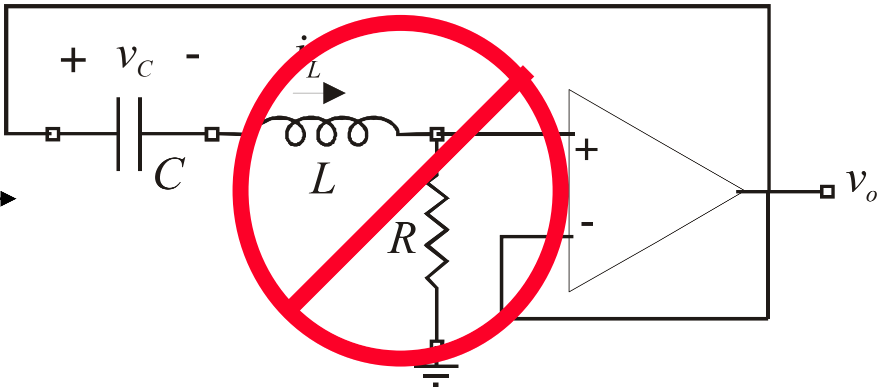
Resonators, Oscillators and Limit Cycles

- **Resonator: a *passive* element that exhibits underdamped oscillatory behavior**
- **Oscillator: a resonator plus an *active* gain element that compensates the resonator losses and results in steady oscillatory behavior**
- **Limiting: a required *nonlinearity* in either the resonator or gain element**
- **Limit Cycle: stable closed path in state space**

Example: Resonant RLC Circuit

- While a linear amplifier can *theoretically* produce an undamped linear system, it cannot create an oscillator





$$\frac{dv_C}{dt} = \frac{1}{C} i_L \quad \longrightarrow \quad \frac{dv_C}{dt} = \frac{1}{C} i_L$$

$$\frac{di_L}{dt} = \frac{1}{L}\left(v_S - v_C - i_L R\right) \quad \longrightarrow \quad \frac{di_L}{dt} = \frac{1}{L}\left(v_0 - v_C - v_0\right) = -\frac{v_C}{L}$$

Example: Resonant RLC Circuit

- No stable limit cycle
- Vary gain A of op-amp circuit

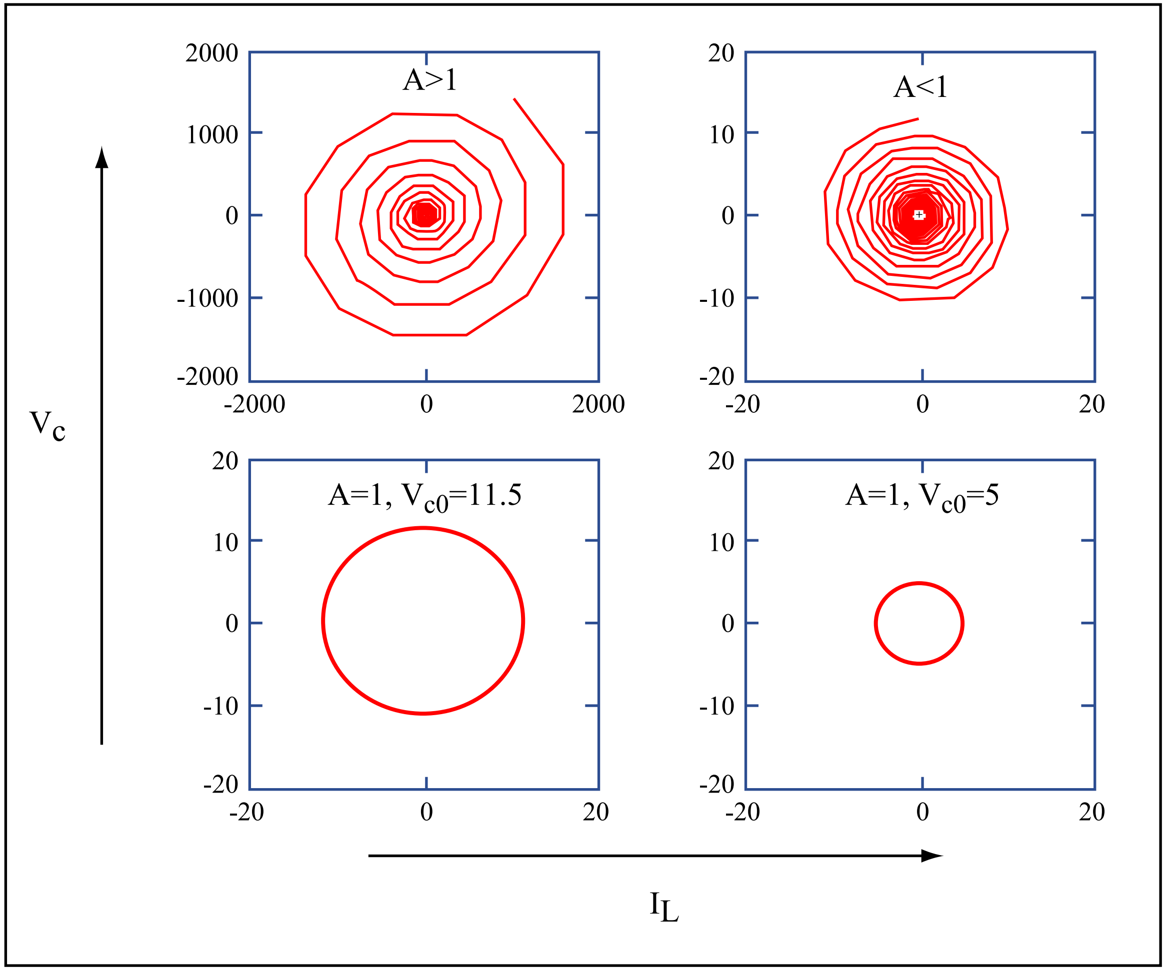


Image by MIT OpenCourseWare.

Adding a limiter creates an oscillator

- **Add in arctangent limiter**

$$v_s = V_1 \tan^{-1}\left(\frac{v_0}{V_2}\right)$$

state eqns.

$$\frac{dv_C}{dt} = \frac{1}{C} i_L$$

$$\frac{di_L}{dt} = \frac{1}{L}\left[V_1 \tan^{-1}\left(\frac{A i_L R}{V_2}\right) - v_C - i_L R\right]$$

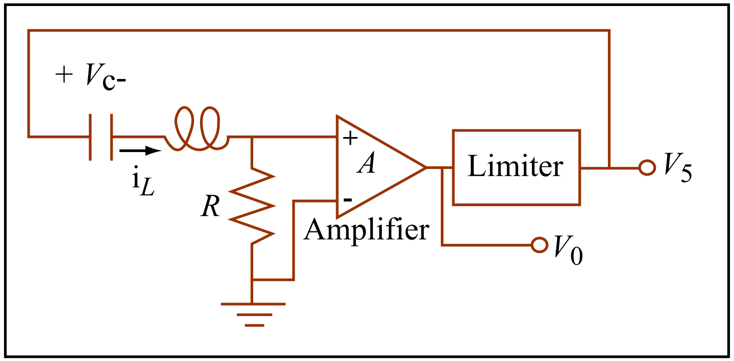
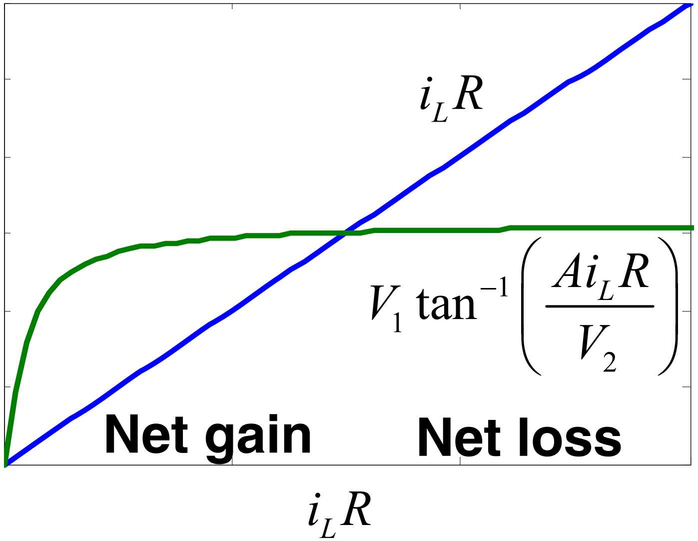


Image by MIT OpenCourseWare.

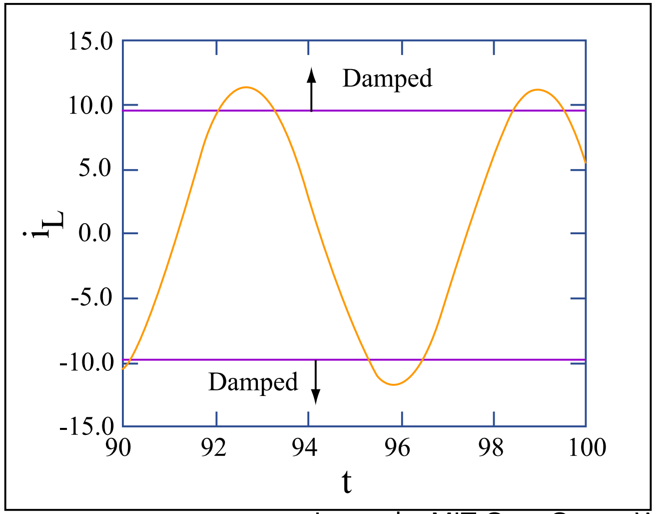
Adapted from Figure 15.15 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 416. ISBN: 9780792372462.

For $\frac{AV_1}{V_2} > 1$



Marginal Oscillator

- Gradual limiting leads to nearly sinusoidal waveforms for weakly damped resonators



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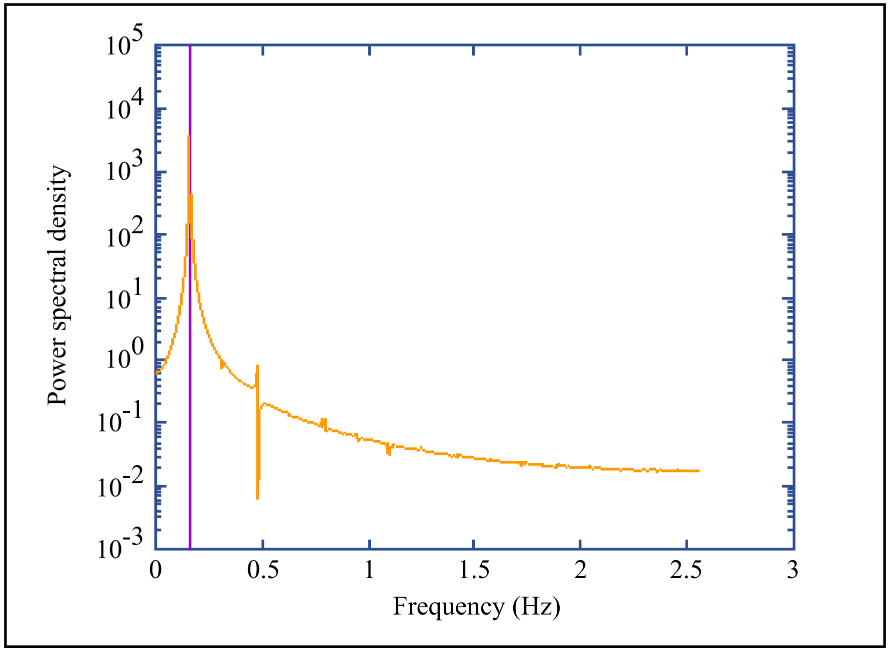
Adapted from Figure 15.17 in: Senturia, Stephen D. *Microsystem Design*

Boston, MA: Kluwer Academic Publishers, 2001, p. 418. ISBN: 97807923724



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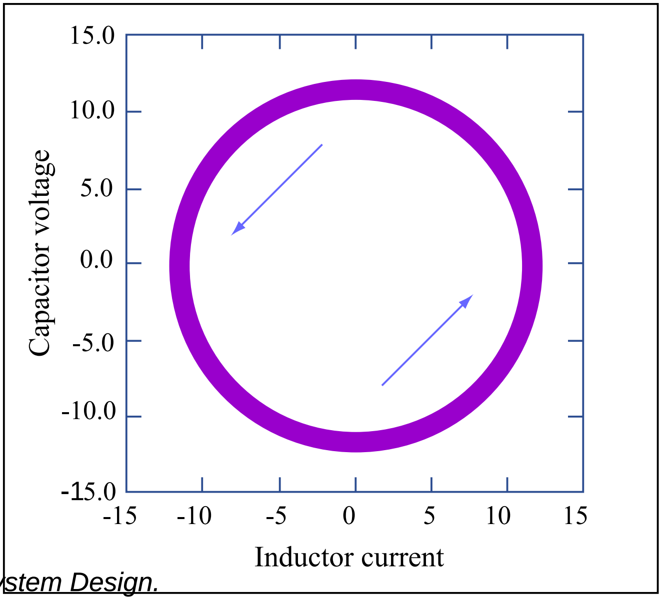

Adapted from Figure 15.18 in: Senturia, Stephen D. *Microsystem Design.*

Boston,MA: Kluwer Academic Publishers, 2001, p. 419. ISBN: 9780792372462.

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Oscillator Parting Comments

- Do not confuse a resonator with an oscillator
- The oscillator is the combined result of a resonator with a suitably designed circuit
- The oscillator is intrinsically nonlinear
- The limit cycle obeys its own dynamics, which can be discovered by analyzing the perturbation of a limit cycle and the time required to recover

Conclusions

- When properly designed, feedback can
  - Reduce sensitivity to variations
  - Decrease response time of system
  - Control output with zero DC error
  - Stabilize unstable systems
- But it may be too complicated or unnecessary for your MEMS part → a systems issue
- All elements in the feedback path have poles, and these can cause instabilities
- Numerous methods exist to analyze control systems in frequency, time, root-locus, and state-space domains