# 6.863J Natural Language Processing Lecture 6: part-of-speech tagging to parsing 

Instructor: Robert C. Berwick

## The Menu Bar

- Administrivia:
- Schedule alert: Lab1 due next today Lab 2, posted Feb 24; due the Weds after this March 5 (web only - can post pdf)
- Agenda:
- Finish up POS tagging - Brill method
- From tagging to parsing: from linear representations to hierarchical representations


## Two approaches

1. Noisy Channel Model (statistical) -
2. Deterministic baseline tagger composed with a cascade of fixup transducers
These two approaches will the guts of Lab 2
(lots of others: decision trees, ...)

## Summary

- We are modeling p(word seq, tag seq)
- The tags are hidden, but we see the words
- Is tag sequence $X$ likely with these words?
- Noisy channel model is a "Hidden Markov Model":
- Find X that maximizes_probability product


## Finding the best path from start to stop



- What is best path from Start to each node?
- Work from left to right
- Each node stores its best path from Start (as probability plus one backpointer)
- Special acyclic case of Dijkstra's shortest-path algorithm
- Faster if somezacas/states are absent


## Method: Viterbi algorithm

- For each path reaching state s at step (word) t, we compute a path probability. We call the max of these viterbi(s,t)
- [Base step] Compute viterbi( 0,0 )=1
- [Induction step] Compute viterbi(s',t+1), assuming we know viterbi(s,t) for all s


## Viterbi recursion

path-prob(s'|s,t) =
probability of path to s' through s

## viterbi(s,t) <br> $a\left[s, s^{\prime}\right]$

transition probability

$$
s \rightarrow s^{\prime}
$$

viterbi $\left(s^{\prime}, t+1\right)=$ max $_{s \in S T A T E S}$ path-prob(s' $\left.\mid \mathrm{s}, \mathrm{t}\right)$

## Viterbi Method...

- This is almost correct...but again, we need to factor in the unigram prob of a state s' emitting a particular word $w$ given an observation of that surface word $w$
- So the correct formula for the path prob to $\mathrm{s}^{\prime}$ from s is: path-prob $\left(s^{\prime} \mid s, t\right)=$ viterbi(s,t) $* a\left[s, s^{\prime}\right] * \quad b_{s^{\prime}}\left(0_{t}\right)$


Bigram
Unigram
Path prob so far to $s$ transition prob output prob at ${ }^{6.8633 / 9.6111}$ Lecture 6 たe®3 state $s^{\prime}$ state s'

## Finally...

- As before, we want to find the max path probability, over all states s:
$\max _{\mathrm{s} \in \text { STATES }}$ path-prob(s' $\mid \mathrm{s}, \mathrm{t}$ )


## Or as in your text...p. 179

function Viterbi(observations of len T,state-graph) returns best-path
num-states $\leftarrow$ NUM-OF-STATES (state-graph)
Create a path probability matrix viterbi[num-states $+2, T+2$ ]
viterbi[0,0] $\leftarrow 1.0$
for each time step $t$ from 0 to $T$ do
for each state $s$ from 0 to num-states do
for each transition $s^{\prime}$ from $s$ specified by state-graph $\underline{\text { new-score } \leftarrow \text { viterbi }[s, t] * a\left[s, s^{\prime}\right] * b_{c}\left(o_{t}\right) \text { Find the path probability }}$ if $\left(\left(\right.\right.$ viterbi $\left.\left[s^{\prime}, t+1\right]=0\right) \|\left(\right.$ new-score $\left.\left.>\operatorname{viterbi}\left[s^{\prime}, t+1\right]\right)\right)$
then

$$
\begin{aligned}
& \text { viterbi }\left[s^{\prime}, t+l\right] \leftarrow \text { new-score } \\
& \text { back-pointer }\left[s^{\prime}, t+l\right] \leftarrow s
\end{aligned}
$$

Find the max so far

Backtrace from highest probability state in the final column of viterbi[] and return path

## Two approaches

1. Noisy Channel Model (statistical) what's that?? (we will have to learn some statistics)
2. Deterministic baseline tagger composed with a cascade of fixup transducers
These two approaches will the guts of Lab 2
(lots of others: decision trees, ...)

# Fixup approach: Brill tagging (a kind of transformation-based learning) 

## Another FST Paradigm: Successive Fixups

- Like successive markups but alter
- Morphology
- Phonology
- Part-of-speech tagging

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## Transformation-Based Tagging (Brill 1995)



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## Brill Tagger

```
Powered by \(\mu\)－TBL Technology
C Swedish © English Russian
Text：
Secretariat is expected to race tomorrow
V Trace
```

```
Analyze
```

```
Analyze
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    Bookmarks & Location: http://www.ling.gu.se/~lager/tagger.cgi?language=English&input=Secretariat+is+expected+to+race+tomorow&<trace=on
```


## Tokenization

```
Secretariat is expected to race tomorrow
```


## Lexical lookup

```
Secretariat/NNP is/VBZ expected/VBN to/TO race/NN tomorrow/NN
```


## Guessing

## Contextual－rule application

Intermediate analysis：

Secretariat／NNP is／VBZ expected／VBN to／TO race／NH tomorrow／NN
Applied rule：
tag：NN＞VB＜－tag：TO＠［－1］．

## Analysis

## Transformation based tagging

- Combines symbolic and stochastic approaches: uses machine learning to refine its tags, via several passes
- Analogy: painting a picture, use finer and finer brushes - start with broad brusch that covers a lot of the canvas, but colors areas that will have to be repainted. Next layer colors less, but also makes fewer mistakes, and so on.
- Similarly: tag using broadest (most general) rule; then an narrower rule, that changes a smaller number of tags, and so on. (We haven't said how the rules are learned)
- First we will se.e. ho3,9.011 tecture the TBL rules are applied


## Applying the rules

1. First label every word with its most-likely tag (as we saw, this gets $90 \%$ right...!) for example, in Brown corpus, race is most likely to be a Noun: $P(\mathrm{NN} \mid$ race $)=0.98$
$P($ VB $\mid$ race $)=0.02$
2. ...expected/VBZ to/T TO race/VB morrow/NN ...the/DT race/NN for/IN outer/JJ space/NN
3. Use transformational (learned) rules to change tags:
Change NN to VB when the previous tag is TO

## Initial Tagging of OOV Words

|  | Change Tag |  |  |
| :---: | :---: | :---: | :---: |
| $\#$ | From | To | Condition |
| 1 | NN | NNS | Has suffix -s |
| 2 | NN | CD | Has character - |
| 3 | NN | JJ | Has character - |
| 4 | NN | VBN | Has suffix -ed |
| 5 | NN | VBG | Has suffix -ing |
| 6 | $? ?$ | RB | Has suffix -ly |
| 7 | $? ?$ | JJ | Adding suffix -ly results in a word. |
| 8 | NN | CD | The word \$ can appear to the left. |
| 9 | NN | JJ | Has suffix -al |
| 10 | NN | VB | The word would can appear to the left. |
| 11 | NN | CD | Has character 0 |
| 12 | NN | JJ | The word be can appear to the left. |
| 13 | NNS | JJ | Has suffix -us |
| 14 | NNS | VBZ | The word it can appear to the left. |
| 15 | NN | JJ | Has suffix -ble |
| 16 | NN | JJ | Has suffix -ic |
| 17 | NN | CD | Has character $\mathbf{1}$ |
| 18 | NNS | NN | Has suffix -ss |
| 19 | $? ?$ | JJ | Deleting the prefix un- results in a word |
| 20 | NN | JJ | Has suffix -ive |

## (JupCi viدcu) ICaililiy kuuuiliy -

How?

- 3 stages

1. Start by labeling every word with most-likely tag
2. Then examine every possible transformation, and selects one that results in most improved tagging
3. Finally, re-tags data according to this rule
4. Repeat 1-3 until some stopping criterion (no new improvement, or small improvement)

- Output is ordered list of transformations that constitute a tagging procedure


## How this works

- Set of possible 'transforms' is infinite, e.g., "transform NN to VB if the previous word was MicrosoftWindoze \& word braindead occurs between 17 and 158 words before that"
- To limit: start with small set of abstracted transforms, or templates


## Templates used: Change $a$ to $b$ when...

The preceding (following) word is tagged $\mathbf{z}$.
The word two before (after) is tagged $\mathbf{z}$.
One of the two preceding (following) words is tagged $\mathbf{z}$.
One of the three preceding (following) words is tagged $\mathbf{z}$.
The preceding word is tagged $\mathbf{z}$ and the following word is tagged $\mathbf{w}$.
The preceding (following) word is tagged $\mathbf{z}$ and the word two before (after) is tagged $\mathbf{w}$.

Variables $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{z}, \boldsymbol{w}$, range over parts of speech

## Method

1. Call Get-best-transform with list of potential templates; this calls
2. Get-best-instance which instantiates each template over all its variables (given specific values for where we are)
3. Try it out, see what score is (improvement over known tagged system -- supervised learning); pick best one locally
function TBL(corpus) returns transforms-queue
IntiALIZE-WITH-MOST-LIKELY-TAGS(corpus)
until end condition is met do
templates $\leftarrow$ Generate-Potential-RELEVANT-TEMPLATES best-transform $\leftarrow$ GET-BEST-TRANSFORM(corpus, templates) APPLY-TRANSFORM(best-transform, corpus)
ENQUEUE(best-transform-rule, transforms-queue)
end
return(transforms-queue)
function GET-BEST-TRANSFORM(corpus, templates) returns transform for each template in templates
(instance, score) $\leftarrow$ GET-BEST-INSTANCE (corpus, template)
if (score $>$ best-transform.score) then best-transform $\leftarrow$ (instance, score) return(best-transform)
function Get-Best-Instance(corpus, template) returns transform
for from-tag $\leftarrow$ from $\operatorname{tag}-1$ to $\operatorname{tag}-n$ do
for $t o-t a g \leftarrow$ from tag -1 to tag $-n$ do
for pos $\leftarrow$ from 1 to corpus-size do
if $($ correct-tag $($ pos $)==$ to-tag \& \& current-tag $($ pos $)==$ from-tag $)$ num-good-transforms(current-tag(pos-1))++ elseif (correct-tag(pos)==from-tag \&\& current-tag(pos)==from-tag) num-bad-transforms(current-tag(pos-1))++
end
best-Z $\leftarrow \operatorname{ARGMAX}_{t}($ num-good-transforms $(\mathrm{t})$ - num-bad-transforms $(\mathrm{t})$,
if(num-good-transforms(best-Z) - num-bad-transforms(best-Z)
$>$ best-instance. Z) then
best-instance $\leftarrow$ "Change tag from from-tag to to-tag
if previous tag is best-Z'
return(best-instance)
procedure APPLY-TRANSFORM(transform, corpus)
for pos $\leftarrow$ from 1 to corpus-size do
if (current-tag $($ pos $)==$ best-rule-from $)$
\&\& $($ current-tag $($ pos -1$)==$ best-rule-prev $))$
current-tag $($ pos $)=$ best-rule-to

## nonlexicalized rules learned by TBL tagger

|  | Change tags |  |  |
| :--- | :--- | :--- | :--- |
| $\#$ | From | To | Condition |
| 1 | NN | VB | Previous tag is TO |
| 2 | VBP | VB | One of the previous 3 tags is MD |
| 3 | NN | VB | One of the previous 2 tags is MD |
| 4 | VB | NN | One of the previous 2 tags is DT |
| 5 | VBD | VBN | One of the previous 3 tags is VBZ |

Example
to/TO race/NN $\rightarrow$ VB
might/MD vanish/VBP $\rightarrow$ VB
might/MD not reply/NN $\rightarrow$ VB

## Transformations Learned

|  | Change Tag |  |  |
| :---: | :---: | :---: | :---: |
| $\#$ | From | To | Condition |
| 1 | NN | VB | Previous tag is $T O$ |
| 2 | VBP | VB | One of the previous three tags is $M D$ |
| 3 | NN | VB | One of the previous two tags is $M D$ |
| 4 | VB | NN | One of the previous two tags is $D T$ |
| 5 | VBD | VBN | One of the previous three tags is $V B Z$ |
| 6 | VBN | VBD | Previous tag is $P R P$ |
| 7 | VBN | VBD | Previous tag is $N A P$ |
| 8 | VBD | VBN | Previous tag is $V B D$ |
| 9 | VBP | VB | Previous tag is $T O$ |
| 10 | POS | VBZ | Previous tag is $P R P$ |
| 11 | VB | VBP | Previous tag is $N N S$ |
| 12 | VBD | VBN | One of previous three tags is $V B P$ |
| 13 | IN | WDT | One of next two tags is $V B$ |
| 14 | VBD | VBN | One of previous two tags is $V B$ |
| 15 | VB | VBP | Previous tag is $P R P$ |
| 16 | IN | WDT | Next tag is $V B Z$ |
| 17 | IN | DT | Next tag is $N N$ |
| 18 | JJ | NNP | Next tag is $N N P$ |
| 19 | IN | WDT | Next tag is $V B D$ |
| 20 | JJR | RBR | Next tag is $J J$ |

BaselineTag*<br>NN @ $\rightarrow$ VB // TO<br>VBP @ $\rightarrow$ VB // ... etc.<br>\section*{Compose this cascade of FSTs.}

## Get a big FST that does the initial tagging and the sequence of fixups "all at once."

## Error analysis: what's hard for

 taggers- Common errors (> 4\%)
- NN vs .NNP (proper vs. other nouns) vs. JJ (adjective): hard to distinguish prenominally; important to distinguish esp. for information extraction
- RP vs. RB vs IN: all can appear in sequences immed. after verb
- VBD vs. VBN vs. JJ: distinguish past tense, past participles (raced vs. was raced vs. the out raced horse)


## What's hard

- Unknown words
- Order 0 idea: equally likely over all parts of speech
- Better idea: same distribution as 'Things seen once' estimator of 'things never seen' - theory for this done by Turing (again!)
- Hapax legomenon
- Assume distribution of unknown words is like this
- But most powerful methods make use of how word is spelled
- See file in the course tagging dir on this


## Or unknown language

- Vse schastlivye sen'i pokhozhi brug na druga, kazhdaja neschastlivaja sem'ja neschastliva po-svoemu


## Most powerful unknown word detectors

- 3 inflectional endings (-ed, -s, -ing); 32 derivational endings (-ion, etc.); capitalization; hyphenation
- More generally: should use morphological analysis! (and some kind of machine learning approach)
- How hard is this? We don't know - we actually don't know how children do this, either (they make mistakes)


## Laboratory 2

- Goals:

1. Use both HMM and Brill taggers
2. Find errors that both make, relative to genre
3. Compare performance - use of kappa \& 'confusion matrix'
4. All the slings \& arrows of corpora - use Wall Street Journal excerpts, as well as 'switchboard' corpus

## Brown/Upenn corpus tags

|  | Tag | Description | Example | Tag | Description | Example |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CC | Coordin. Conjunction | and, but, or | SYM | Symbol |  |
|  | CD | Cardinal number | one, two, three | TO | "to" | to |
|  | DT | Determiner | a, the | UH | Interjection | ah, oops |
|  | EX | Existential 'there' | there | VB | Verb, base form | eat |
|  | FW | Foreign word | mea culpa | VBD | Verb, past tense | ate |
|  | IN | Preposition/sub-conj | of, in, by | VBG | Verb, gerund | eating |
|  | JJ | Adjective | yellow | VBN | Verb, past participle | eaten |
|  | JJR | Adj., comparative | bigger | VBP | Verb, non-3sg pres | eat |
|  | JJS | Adj., superlative | wildest | VBZ | Verb, 3sg pres | eats |
| ] text | LS | List item marker | 1, 2, One | WDT | Wh-determiner | which, that |
| J. textı | MD | Modal | can, should | WP | Wh-pronoun | what, who |
| D. 297 | NN | Noun, sing. or mass | llama | WP\$ | Possessive wh- | whose |
| 1. 297 | NNS | Noun, plural | llamas | WRB | Wh-adverb | how, where |
| 98.6 | NNP | Proper noun, singular | IBM | \$ | Dollar sign | \$ |
| 1M words | NNPS | Proper noun, plural | Carolinas | \# | Pound sign | \# |
| 60K tag | PDT | Predeterminer | all, both | " | Left quote | (' or ") |
| 60K tag | POS | Possessive ending | 's | " | Right quote | (' or ") |
| counts | PP | Personal pronoun | I, you, he | ( | Left parenthesis | $\left(\left[,\left(,,^{\prime},<\right)\right.\right.$ |
|  | PP\$ | Possessive pronoun | your, one's | ) | Right parenthesis | ( ], ), \}, >) |
|  | RB | Adverb | quickly, never |  | Comma |  |
|  | RBR | Adverb, comparative | faster |  | Sentence-final punc | (. ! ? ) |
|  | RBS | Adverb, superlative | fastest |  | Mid-sentence punc | $(: ; \ldots-)$ |
|  | RP | Particle | up, off |  |  |  |

## Coda on kids

C: "Mommy, nobody don't like me"
A: No, say, "nobody likes me"
C: Nobody don't likes me
A: Say, "nobody likes me"
C: Nobody don't likes me
[ 7 repetitions]
C: Oh! Nobody don't like me!
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## Parsing words - review

- We are mapping between surface, underlying forms
- Sometimes, information is 'invisible' (I.e., erased $e$, or an underlying/surface 0 )
- There is ambiguity (more than one parse)


## From lines to hierarchical respresentions...

- From this:
morph-ology
- To this:

VP [head=vouloir,...]

$\mathbf{V}_{\text {[head=}}=$ vouloir, ...
the problem of morphology $\longrightarrow$ veut ("word shape") -
tense=Present,
num=SG, person=P3]
veut

## What can't linear relations represent?

- wine dark sea $\rightarrow$ (wine (dark sea)) or ((wine dark) sea) ?
- deep blue sky
- Can fsa's represent this?
- Not really: algebraically, defined as being associative (doesn't matter about concatenation order)


## So, from linear relations... to hierarchies



## Examples

Verb $\rightarrow$ thrills
VP $\rightarrow$ Verb NP
$S \rightarrow N P V P$

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# Parsing for fsa's: keep track of what 'next state' we could be in 

 at each step

NB: ambiguity $=>1$ path through network

$$
\begin{aligned}
& =>1 \text { sequence of states ('parses') }
\end{aligned}
$$

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## Brill Tagger

Powered by $\mu$－TBL Technology
C Swedish © English $\subset$ Russian
Text：


V Trace Analyze

## Tokenization

fruit flies like a banana

## Lexical lookup

fruit／NN flies／VBZ like／IN a／DT banana／NN

## Guessing

## Contextual－rule application

## FSA Terminology

- Transition function: next state unique = deterministic fsa
- Transition relation: > 1 next state $=$ nondeterministic fsa



## Methods for parsing

- How do we handle ambiguity?
- Methods:

1. Backtrack
2. Convert to deterministic machine (ndfsa $\rightarrow$ dfsa): offline compilation
3. Pursue all paths in parallel: online computation ("state set" method)
4. Use lookahead

- We will use all these methods for more complex machines/language representations


## FSA terminology

- Input alphabet, $\Sigma$; transition mapping, $\delta$; finite set of states, $Q$; start state $q_{0}$; set of final states, $\mathrm{q}_{\mathrm{f}}$
- $\delta(q, s) \rightarrow q^{\prime}$
- Transition function: next state unique = deterministic fsa
- Transition relation: > 1 next state = nondeterministic fsa


## State-set method: simulate a nondeterministic fsa

- Compute all the possible next states the machine can be in at a step = state-set
- Denote this by $S_{i}=$ set of states machine can be in after analyzing $i$ tokens
- Algorithm has 3 parts: (1) Initialize; (2) Loop; (3) Final state?
- Initialize: $S_{0}$ denotes initial set of states we're in, before we start parsing, that is, $q_{0}$
- Loop: We must compute $S_{i}$, given $S_{i-1}$
- Final?: $S_{f}=$ set of states machine is in after reading all tokens; we want to test if there is a final state in sthere eatue 5 sol3


## State-set parsing

## Initialize: $\quad$ Compute initial state set, $\mathrm{S}_{0}$

1. $\mathrm{S}_{0} \leftarrow \mathrm{q}_{0}$
2. $\mathrm{S}_{0} \leftarrow \varepsilon$-closure $\left(\mathrm{S}_{0}\right)$

Loop: $\quad$ Compute $S_{i}$ from $S_{i-1}$

1. For each word $\mathrm{w}_{\mathrm{i}}, \mathrm{i}=1,2, \ldots, \mathrm{n}$
2. $S_{i} \leftarrow \bigcup_{q \in S_{i-1}} \delta\left(q, w_{i}\right)$
3. $\mathrm{S}_{\mathrm{i}} \leftarrow \varepsilon$-closure $\left(\mathrm{S}_{\mathrm{i}}\right)$
4. if $S_{i}=\varnothing$ then halt $\&$ reject else continue

Final: Accept/reject


## What's the minimal data

## structure we need for this?

- [ $S, i$ ] where $S=$ denotes set of states we could be in; $i$ denotes current point we're at in sentence
- As we'll see, we can use this same representation for parsing w/ more complex networks (grammars) - we just need to add one new piece of information for state names
- In network form

- In rule form:
$q_{i} \rightarrow t \bullet \beta q_{f} \quad$ where $\tau=$ some token of the input, and $\beta=$ remainder (so 'dot' represents how far we have traveled)
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## Example


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## Use backpointers to keep track of the different paths (parses):



## When is it better to convert at compile time vs. run time? (for fsa)

- Run time: compute next state set on the fly
- Compile time: do it once and for all
- When would this difference show up in natural languages (if at all)?


## Where do the fsa states come from?

- States are equivalence classes of words (tokens) under the operation of substitution
- Linguistic formulation (Wells, 1947, pp. 8182): "A word $A$ belongs to the class determined by the environment ___ $X$ if $A X$ is either an utterance or occurs as a part of some utterance" (distributional analysis)
- This turns out to be algebraically correct
- Can be formalized - the notion of syntactic equivalenceras3,.611 Letwre 5 spo3


## X-files: fragments from an alien

 language1. Werid lost the election
2. Gore will lose the election
3. Gore could lose the election
4. Gore should lose the election
5. Gore did lose the election
6. Gore could have lost the election
7. Gore should have lost the election
8. Gore will have lost the election
9. Gore could have been losing the election
10. Gore should have been losing the election
11. Gore will have been losing the election
12. Gore has lost the election

## More X-files

14. Bush lost the election
15. Bush will lose the election
16. Bush could lose the election
17. Bush should lose the election
18. Bush did lose the election
19. Bush could have lost the election
20. Bush should have lost the election
21. Bush will have lost the election
22. Bush could have been losing the election
23. Bush should have been losing the election
24. Bush will have been losing the election
25. Bush has .ost the eleation

## Formally...

- Definition. A binary relation between sets $A, B$, is a subset (possibly empty) of $A \times B$
- Definition. Strings $k, r$ are left-substitutable in a language $L$, if, for all strings $w$ defined over $\Sigma^{*}$, $k w \in L$ iff $r w \in L$
- Fact. Left-substitutability is an equivalence relation (reflexive, transitive, symmetric)
- Definition. An equivalence relation over $\Sigma$ is finite rank if it divides $\Sigma$ into finitely many equivalence classes
- Definition. A binary relation $R$ is called rightinvariant if, for all $p, r \in \Sigma^{*}, p R r \Rightarrow p w R r w$


## And formally...

- Fact. A right-invariant relation $R$ is an equivalence relation
- Theorem (Myhill-Nerode, 1956)


## Theorem (Myhill-Nerode, 1956).

- Let $L \subseteq \Sigma^{*}$. Then the following 3 propositions are equivalent:

1. L is generated (accepted) by some finitestate automaton (finite transition network);
2. $L$ is the union of certain equivalence classes of a right-invariant equivalence relation of finite rank
3. Let the equivalence relation $R$ be defined as follows: $x R y$ iff $x$ and $y$ are left-substitutable in $L$. Then this relation $R$ is of finite-rank and is right-invariant [this is Wells' definition]

## Finite \# of bins = finite state

Gives easy way to show what is not finite-state
Eg, $a^{n} c b^{n}$, for all $n>0$
Proof by contradiction.
Suppose there was such an FSA. By the theorem, this FSA is of finite rank, and classifies all strings in $\Sigma^{*}$ into one of a finite number of classes.
By the pigeonhole principle, there must exist some string $a^{i}$ s.t. $a^{j}$ with $j \neq i$ is in the same equivalence class as $a^{i}$. But then the fsa must recognize both $a^{i} c a^{j}$ and $a^{i} \mathrm{c} a^{i}$, a contradiction

## Why not fsa's forever?

- Can't yield the right set of strings= weak generative capacity (antiantimissle...)
- Can't yield the right set of structures = strong generative capacity (dark blue sky)
- How do these failures show up?


## A more complex fsa



## Conversion to deterministic machine


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## What are we missing here?

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## We are missing the symmetry



## Having a poor representation...

- Shows up in having duplicated states (with no other connection to each other)
- System would be 'just as complex'= have the same size (what is size of automaton?) even if the network were not symmetric
- So we have failed to capture this regularity \& the network could be compressed
- How?


## Compressability reveals rendundancy (pattern)that we have missed

Active:

$+$
Rule that flips network=

Passive:


Aka "transformational grammar"

# But it's worse than that... more redundancy even so 



So, obvious programming approach: use a subroutine

## Subnetworks as subroutines, to compress the description




Noun phrase:


Bush
"splice out" common subnets

## Could be worse...

## Could be raining...



Noun "specifiers"

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## It could be even worse...



## Examples

Verb $\rightarrow$ thrills
VP $\rightarrow$ Verb NP
$S \rightarrow N P V P$

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## The notion of a common subnetwork

- Equivalent to the notion of a phrase
- A Noun Phrase (NP)
- Defined by substitution class of a sequence of words (aka "a constituent") - extension beyond substitution of single words
- A phrase iff we can interchangeably substitute that sequence of words regardless of context
- So also gives us the notion of a context-free grammar (CFG)


## Constituents, aka phrases

- Building blocks that are units of words concatenated together
- Why?
- Ans:

1. They act together (i.e., behave alike under operations) - what operations?
2. Succinctness
3. (Apparently) nonadjacent constraints

## The deepest lesson

- Claim: all apparently nonadjacent relationships in languge can be reduced to adjacent ones via projection to a new level of representation
- (In one sense, vacuous; in another, deep)
- Example: Subject-Verb agreement (agreement generally)
- Example: so-called wh-movement


## Gaps ("deep" grammar!)

- Pretend "kiss" is a pure transitive verb.
- Is "the president kissed" grammatical?
- If so, what type of phrase is it?
- the sandwich that
- I wonder what
- What else has
the president kissed e
Sally said the president kissed e
Sally consumed the pickle with e
Sally consumed e with the pickle


## Examples

- The guy that we know in Somerville likes icecream
- Who did the guy who lives in Somerville see



## The deep reason why

- Machinery of the mind: based only on concatenation of adjacent elements - not on 'counting' eg., "take the 7th element \& move it..."
- Runs through all of linguistic representations (stress, metrical patterns, phonology, syntax, ...)
- Strong constraint on what we have to represent


## Constituents

- Basic 'is-a' relation
- Act as 'whole units' -
- I want this student to solve the problem
- ?? Student, I want this to solve the problem
- This student, I want to solve the problem
- Sometimes, we don't see whole constituents...book titles (claimed as objection to constituency):
- Sometimes a Great Notion
- The Fire Next Time
- Why might that be?

