Multidisciplinary System Design Optimization (MSDO)

How to Break "Robust" Optimizers

Recitation 6

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Today's Topics

- fmincon
  - What the options are
  - What it's doing
  - How to break it
- GA toolbox on Stellar
- MATLAB® GA Toolbox
  - Some options
  - How to break it
- Questions

Sequential Quadratic Programming

- Create Quadratic Approximation: $B(\mathbf{x}_k) \approx \nabla_{xx} L(\mathbf{x}_k); \quad B(\mathbf{x}_k) \succ 0$

$$s_k = x_{k+1} - x_k$$

$$q_k = \left(\nabla f(x_{k+1}) + \sum_{i=1}^{m_{ineq}} \lambda_i \nabla g_i(x_{k+1}) + \sum_{j=1}^{m_{eq}} \lambda_j \nabla h_j(x_{k+1}) \right) - \left(\nabla f(x_k) + \sum_{i=1}^{m_{ineq}} \lambda_i \nabla g_i(x_k) + \sum_{j=1}^{m_{eq}} \lambda_j \nabla h_j(x_k) \right)$$

$$B_{k+1} = B_k + \frac{q_k q_k^T}{q_k^T s_k} - \frac{B_k^T s_k^T s_k B_k}{s_k^T B_k s_k}$$

- Solve Quadratic Program:

$$\min_{d \in \Re^n} \frac{1}{2} d^T B_k d + \nabla f(x_k)^T d$$
$$s.t. \quad A_{eq} d = b_{eq}$$
$$\bar{A} d \le \bar{b}$$

- Evaluate merit function: $\Psi(x) = f(x) + \sum_{i=1}^{m_{eq}} r_i h_i(x) + \sum_{i=1}^{m_{ineq}} r_i \cdot \max[g_i(x), 0]$
- Compute step length: $\alpha_k = \arg \min_{\alpha \in A(\alpha d) \le b} \Psi(x_k + \alpha d_k)$
- Take step: $x_{k+1} = x_k + \alpha_k d_k$
- Repeat until $\alpha_k d_k \le \varepsilon$

Interior Point Algorithm

- Two equivalent optimization problems:

$$\begin{aligned}&\min_{x,s\in\Re^n} f(x)\\ &s.t.\quad h(x)=0\\ &\qquad\ g(x)+s=0\\ &\qquad\ s\geq 0\end{aligned}
\qquad
\begin{aligned}&\min_{x,s\in\Re^n} f(x)-\mu\sum_{i=1}^{m_{ineq}}\log s_i\\ &s.t.\quad h(x)=0\\ &\qquad\ g(x)+s=0\end{aligned}$$

- KKT Conditions:

$$\begin{aligned}&\nabla f(x)+\sum_{i=1}^{m_{ineq}} z_i\nabla g_i(x)+\sum_{j=1}^{m_{eq}} y_j\nabla h_j(x)\\ &sz-\mu e=0\\ &h(x)=0\\ &g(x)+s=0\end{aligned}
\qquad
\begin{aligned}&\nabla f(x)+\sum_{i=1}^{m_{ineq}} z_i\nabla g_i(x)+\sum_{j=1}^{m_{eq}} y_j\nabla h_j(x)\\ &-\mu S^{-1}e+z=0\\ &h(x)=0\\ &g(x)+s=0\end{aligned}$$

- Solve KKT system with Newton's method, for directions.
- Compute step length: (τ~0.995)

$$\alpha_s=\max\{0<\alpha\leq 1: s+\alpha d_s\geq(1-\tau)s\}$$
$$\alpha_z=\max\{0<\alpha\leq 1: z+\alpha d_z\geq(1-\tau)z\}$$

- Take step:

$$x_{k+1}=x_k+\alpha_s d_x \qquad s_{k+1}=s_k+\alpha_s d_s$$
$$y_{k+1}=y_k+\alpha_z d_y \qquad z_{k+1}=z_k+\alpha_z d_z$$

- Repeat until KKT Conditions satisfied to within μ_k
- Repeat entire process with $\mu_{k+1}=\sigma\mu_k$,$0<\sigma<1$

Genetic Algorithm

16.888
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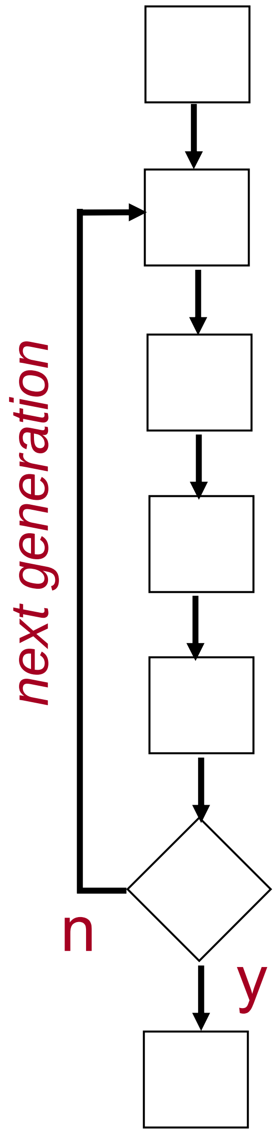

Initialize Population (initialization)

Select individual for mating (selection)

Mate individuals and produce children (crossover)

Mutate children (mutation)

Insert children into population (insertion)

Are stopping criteria satisfied ?

Finish

Ref: Goldberg (1989)

GA Toolbox

- BUG!!!
- Add into genetic.m
  - Line 118: stats=[];

Fitness Function

- Only has Roulette Wheel selection
  - You can add others…
- Roulette Wheel

$$P = \frac{f\left(\mathbf{x}_k\right)}{\sum_k f\left(\mathbf{x}_k\right)}$$

- Are there any restrictions on f(x)?

Matlab GA Toolbox

- Constraint Handling
- Augmented Lagrangian/Penalty method:

$$\min_{x \in \Re^n} f(x) - \sum_{i=1}^{m_{ineq}} \lambda_i s_i \log\left(g_i(x) - s_i\right)$$

$$g_i(x) - s_k < 0$$

$$s_k = \frac{1}{\mu_k} \lambda_k$$

  – $\mu_0 \geq 1$
  – $\sigma > 1$; $\mu_{k+1} = \sigma \mu_k$
- Disclaimer: this is one of three papers referenced, it may not be exactly what's in MATLAB

Summary

- There are many optimization toolboxes available to you.
- Everyone of them has limitations.
- Know what they are doing.

- Assure your problem fits within the assumptions that algorithm makes!

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