## Less

## 1. Summary from last week

(1) adjective meanings: relational (Rullmann's 'at least' interpretation), i.e.
$[[$ tall $]]=\square \mathrm{d} . \square \mathrm{x} . \operatorname{TALL}(\mathrm{x}) \geq \mathrm{d}$
Functional ('exactly') meanings were equivalent for almost all purposes, but didn't seem to work with necessity modals.
(2) meaning of the comparative morpheme -er (= more):
$\square_{\text {matrix-clause }}\left[-e r \square_{\text {than-clause }}\right]$ is true iff
some degree that satisfies $\square \quad>$ every degree that satisfies $\square$ the maximal the maximal

In infinite, continuous domains, 'some - every' seemed superior, but otherwise the maximality versions were equivalent.

Another variant which is practically always equivalent:
(3) $\quad \square_{\text {matrix-clause }}\left[\right.$-er $\left.\square_{\text {than-clause }}\right]$ is true iff the set of degrees that satisfy $\square \quad$ the set of degrees that satisfy $\square$ (i.e., is a proper superset of)

Equatives:
(4) $\quad \square_{\text {matrix-clause }}\left[(\right.$ at least $)$ as $\left.\square_{\text {as-clause }}\right]$ is true iff some degree that satisfies $\square \geq$ every degree that satisfies $\square$ the maximal the maximal the set of degrees that satisfy $\square \supseteq$ the set of degrees that satisfy $\square$
(5) $\quad \square_{\text {matrix-clause }}\left[\right.$ exactly as $\left.\square_{\text {as-clause }}\right]$ is true iff some degree that satisfies $\square=$ every degree that satisfies $\square$ the maximal the maximal
the set of degrees that satisfy $\square=$ the set of degrees that satisfy $\square$
Possibly an advantage for the set-based formulation emerges here: it seems to combine the virtues of the maximality formulation with the ability to handle continuous domains. However, this is not a compositional analysis of 'exactly as'.
(6) The package is exactly as heavy as it is allowed to be. (cannot be literally true if the post office specifies 'less than 5 lbs ')
(7) The package is allowed to be exactly as heavy as the letter is (allowed to be). (arguably true if PO specifies 'less than 5 lbs ' for packages as well as letters)

Differential comparatives:
(8) $\quad \square_{\text {matrix-clause }}\left[(\right.$ at least $) \mathrm{d}$-er $\left.\square_{\text {than-clause }}\right]$ is true iff
(a) some degree that satisfies $\square$ exceeds every degree that satisfies $\square$ by at least d
(b) the maximal degree that satisfies $\square \geq d+$ the maximal degree that satisfies $\square$
(c) the set of degrees that satisfy $\square \supseteq d+$ the set of degrees that satisfy $\square$
(9) Let A be a set of degrees and d a degree. Then
$\mathrm{A}+\mathrm{d}:=\mathrm{A} \square\left\{\mathrm{d}^{\prime}+\mathrm{d}: \mathrm{d}^{\prime} \square \mathrm{A}\right\}$
(10) $\quad \square_{\text {matrix-clause }}\left[\right.$ exactly d -er $\left.\square_{\text {than-clause }}\right]$ is true iff
(b) the maximal degree that satisfies $\square=d+$ the maximal degree that satisfies $\square$
(c) the set of degrees that satisfy $\square=d+$ the set of degrees that satisfy $\square$
(11) The package is exactly 1 oz heavier than it is allowed to be.
(12) The package is allowed to be exactly 1 oz heavier than the letter is (allowed to be).

But recall conflicting evidence from 'exactly' equatives and 'exactly' diff. comparatives containing disjunctions.

## 2. less comparatives

### 2.1. Simple cases

(13) Mary is less tall than John is.

Given our decision in favor of relational adj meanings, we seem to need one of the following:
(14) $\quad \square_{\text {matrix-clause }}\left[\right.$ less $\left.\square_{\text {than-clause }}\right]$ is true iff
(a) every degree that satisfies $\square<$ some degree that satisfies $\square$
(b) the maximal degree that satisfies $\square<$ the maximal degree that satisfies $\square$
(c) the set of degrees that satisfy $\square \quad \square$ the set of degrees that satisfy $\square$

Rullmann: less comparatives argue for

- maximality analysis of than-clauses, and
- functional adj meanings
(15) Rullmann's meanings for more and less:
$\square_{\text {matrix-clause }}\left[\right.$-er $\left.\square_{\text {than-clause }}\right]$ is true iff
some degree that satisfies $\square>$ the maximal degree that satisfies $\square$
$\square_{\text {matrix-clause }}\left[\right.$ less $\left.\square_{\text {than-clause }}\right]$ is true iff
some degree that satisfies $\square<$ the maximal degree that satisfies $\square$
He assumes that the quantificational force of the than-clause comes from the semantics of the wh-clause itself (a kind of free-relative analysis). Hence this should be uniform across different degree operators.

As for the quantiifcational force in the matrix, he takes it to be contributed by the degree operator. Thus no real reason for uniformity (e.g., 'some' throughout). So he could have chosen (14b) and relational adj meanings instead - at least for what we have seen so far.
2.2. Two readings with less

Sentences for which the analysis so far is adequate (from Rullmann):
(20) John is less strong than he could be.
(21) Students spend less than a professor can spend.

However, we make incorrect predictions for the following examples:
(22) John is less tall than any of the girls.
predicted: *'he is shorter than the tallest girl'
observed: 'he is shorter than the shortest girl'
(23) John is less tall than he ever was.
predicted: *'he is below his greatest previous height'
observed: 'he is below his lowest previous height'
(24) amended meaning suggested by these examples (see Rullmann):
$\square_{\text {matrix-clause }}\left[\right.$ less $\left.\square_{\text {than-clause }}\right]$ is true iff
some (the maximal) degree that satisfies $\square<$ the minimal degree that satisfies $\square$
This, however, crucially relies on using functional adj meanings after all! Reference to minima otherwise does not make any sense.

Rullmann's findings:

- less is ambiguous between 'below minimum' and 'below maximum' readings:
(25) $\quad\left[\right.$ less $\left._{<\text {max }} \square_{\text {than-clause }}\right] \square_{\text {matrix-clause }}$ is true iff some degree that satisfies $\square<$ the maximal degree that satisfies $\square$
(26) $\quad\left[\right.$ less $\left._{<\min } \square_{\text {than-clause }}\right] \square_{\text {matrix-clause }}$ is true iff some degree that satisfies $\square<$ the minimal degree that satisfies $\square$
- ambiguity emerges only with quantifiers:
(27) When there is no quantifier (incl. time or world quantifiers) in the than-clause, then minimum $=$ maximum $=$ unique degree that satisfies than-clause. Hence the ambiguity is indetectable.
- NPIs disambiguate:
(28) Only the less $_{<\min }$ reading is available when the than-clause contains negative polarity items. This follows from the logical properties of the two readings together with Ladusaw's theory of NPI-licensing.
(29) I got more than I could ever pay back.
(30) I got less than I could ever live on.
- existence presuppositions for each reading:
(32) Only the less $\boldsymbol{< m i n}$ reading is available when no maximum exists, and only the less $_{<\boldsymbol{m a x}}$ reading is available when no (non-trivial) minimum exists.
(33) The monastery scenario: Praying is good, sleeping is bad.

You are required to pray at least 6 hours a day, and the more you pray, the better.
You are allowed to sleep at most 6 hours a day, and the less you sleep, the better.
(34) I slept less than was allowed.
(true if, e.g., I slept 5 hours; less $_{<\max }$ reading)
(35) I prayed less than was allowed.
(true if, e.g., I prayed for 5 hours; less $_{<\text {min }}$ reading)
Sentences that are ambiguous, because minima as well as maxima exist:
(36) The helicopter was flying less high than a plane can fly.
(a) the h . was flying at an altitude below the maximal altitude at which a plane can fly
(b) the h . was flying at an altitude below the minimal altitude at which a plane can fly
(37) Lucinda is driving less fast than is allowed on this highway. (from Rullmann 1995a)
(a) she's driving below the speed limit
(b) she's driving below the minimum speed

- There is no analogous ambiguity in more-comparatives:
(38) Lucinda is driving faster than is allowed on this highway.
(a) she's driving above the speed limit
(b) *she's driving above the minimum speed
(39) in monastery scenario:
(a) I slept more than was allowed.
(true if, e.g., I slept 7 hours)
(b) \#I prayed more than was allowed.
(cannot be true in this scenario)


## 3. Shortcomings of Rullmann's initial analysis

conceptual issue:
(40) Why is less ambiguous?

Is it an accident that less is ambiguous but more isn't?
empirical issue: apparently similar ambiguities arise elsewhere.
(41) It's amazing how little one is allowed to eat here.
(a) I thought one would have been allowed to eat more.
(b) I thought one would have been required to eat more.
(42) We are shocked that they are allowed to teach so little.
(a) 'we are shocked that they aren't allowed to teach more'
(b) 'we are shocked that they get away with so little teaching'
(43) It's a disgrace how little they are allowed to sleep in this monastery!
(= they should have been allowed to sleep more)
(44) It's a disgrace how little they are allowed to pray in this monastery! (= they should have been required to pray more)

Suspicion: The ambiguity is related to the presence of little. It shows up systematically with expressions of the form [degree morpheme + little], where the degree morpheme can be how, so, -er, etcetera. less is just one special case, the suppletive spell-out of -er little.

## 4. Rullmann's second account of the ambiguity

$$
\begin{align*}
& \text { faster }=\text {-er fast }  \tag{45}\\
& \text { less fast }=\text {-er little fast }
\end{align*}
$$

The ambiguity arises because little can be either part of the comparative operator or part of the adjective.

She drove faster than she was allowed to. -- only one derivation:
she drove [Deg_er] [Adj fast] than wh she was allowed to [VP]
Deg + than-clause takes scope:
[-er than wh she was allowed to _] ${ }_{1}$ she drove $t_{1}$ fast ellipsis filled in contains matching adjective:
[-er than wh $\boldsymbol{w}_{2}$ she was allowed to drive $\mathbf{t}_{\underline{2}}$ fast $_{1}$ she drove $\mathbf{t}_{1}$ fast meaning: 'faster than maximal permitted speed'

She drove less fast than she was allowed to. -- first derivation:
she drove [Deg_er little] [Adj_fast] than wh she was allowed to [VP]
Deg + than-clause takes scope:
[-er little than wh she was allowed to _] she drove $t_{1}$ fast ellipsis filled in contains matching adjective:
[-er little than wh2 she was allowed to drive $t_{2}$ fast $]_{1}$ she drove $t_{1}$ fast
meaning: 'slower than maximal permitted speed'
(48) She drove less fast than she was allowed to. -- second derivation:
she drove [Deg_er] [Adjlittle fast] than wh she was allowed to [vp ]
Deg + than-clause takes scope:
[-er than wh she was allowed to _] ${ }_{1}$ she drove $t_{1}$ little fast
ellipsis filled in contains matching adjective:
[-er than wh $\mathbf{w}_{2}$ she was allowed to drive $\mathbf{t}_{2}$ little fast $]_{1}$ she drove $\mathbf{t}_{\mathbf{1}}$ little fast meaning: 'slower than minimal required speed'

But how exactly do these structures express these respective meanings?
Sketch of semantics
[-er $\left.\square_{\text {than-clause }}\right] \square_{\text {matrix-clause }}$ is true iff some degree that satisfies $\square>$ the maximal degree that satisfies $\square$
(50) [-er little $\left.\square_{\text {than-clause }}\right] \square_{\text {matrix-clause }}$ is true iff some degree that satisfies $\square<$ the maximal degree that satisfies $\square$

Note: this is the same semantics previously given to the less $_{<\boldsymbol{m a x}^{\prime}}$ homonym.
(51) $\quad[[f \mathbf{f a s t}]]=\square \mathrm{d}$ : d is a degree of fastness. $\square \mathrm{x} . \mathrm{x}$ is d-fast
(52) Degrees of fastness are ordered in such a way that higher speeds correspond to greater degrees of fastness.
(53) $\quad[$ little fast $]]=[[$ slow $]]=\square \mathrm{d}$ : d is a degree of slowness. $\square \mathrm{x} . \mathrm{x}$ is d-slow
(54) Degrees of slowness are ordered in such a way that lower speeds correspond to greater degrees of slowness.

Predicted readings:
(55) for (46):

She had a degree of fastness above the maximal allowable fastness.
(56) for (47):

She had a degree of fastness below the maximal allowable fastness.
(57) for (48):

She had a degree of slowness above the maximal allowable slowness.
= She had a degree of fastness below the minimal allowable fastness.
Evaluation of this account
Advantage over previous account (see criticism (40)):
(58) less is now more complex than -er (more), so there is a principled reason why it gives rise to more readings.

But there are still problems:

- compositionality:
(59) There is no meaning for little by itself. Entries are given for -er little in (50), and for little fast in (53). These collocations are, in effect, treated as idioms.
(60) Challenge: define a uniform meaning for little that converts the meaning of -er into the meaning of less <max , and also coverts the meaning of fast into that of slow.
- generality:
(61) To capture the ambiguity in exclamatives like (41) - (44) in an analogous way, we have to posit not just two structures for each sentence, but also separate entries for how and how little.
(62) $\quad\left[D_{\text {gh }}{ }^{2}\right]_{1}$ they are allowed to pray $\mathbf{t}_{1}$ little
(63) [Deghow little $]_{1}$ they are allowed to sleep $\mathbf{t}_{\mathbf{1}}$ much
(64) how $\square$ ! conveys that the maximal degree satisfying $\square$ was expected to be lower.
(65) how little $\square$ ! conveys that the maximal degree satisfying $\square$ was expected to be higher.


## 5. A compositional variant of Rullmann's second account

### 5.1. Syntax: DegP-recursion

structure of simple comparative:
(66) $\quad\left[\mathrm{DegP}\right.$-er than $\left[{ }_{C P} \mathbf{w h} \ldots .\right]_{\mathbf{i}}$ John is $\mathbf{t}_{\mathbf{i}}$ tall
a well-known case of potential recursion in DegP: differential comparatives
(67) (a) John is 6 inches taller than Mary.
[DegP6 inches -er than wh ....] $]_{\mathbf{i}}$ John is $\mathbf{t}_{\mathbf{i}}$ tall
(b) John is as much taller than Mary as you are.
[DegP as as wh .... $]_{\mathbf{j}}$ [ [DegP $\mathbf{t}_{\mathbf{j}}$-er than wh .... $]_{\mathbf{i}} \mathbf{J o h n}$ is $\mathbf{t}_{\mathbf{i}}$ tall $]$
a new case of nested DegPs: little
(68) LF-structure for John is less tall than Mary

(69) content of elided 'than'-clause must match, hence "...." is as follows:
$\mathbf{w h}_{\mathbf{k}}{\left[\text { DegP }_{\mathbf{k}}\right.}$ little] $]_{\mathbf{l}}$ Mary is $\mathbf{t}_{\mathbf{l}}$ tall
comparison with Rullmann:
Like Rullmann, I decompose less into -er and little. Unlike Rullmann, I never treat little as part of the top-most comparative operator. little always stays within the clause that is the comparative operator's nuclear scope. Therefore, it must always be present in the 'than'-clause as well, to ensure the proper parallelism for ellipsis.

### 5.2. Semantics

5.2.1 Background: amended semantics of adjectives and comparative -er
(70) Adjectives: relational ("at least") interpretation
$\llbracket$ tall $\rrbracket=\square \mathrm{d} . \square \mathrm{x} . \operatorname{TALL}(\mathrm{x}) \geq \mathrm{d}$
(73) Example: suppose John is exactly 6 ft tall.

Then $\{\mathrm{d}: \operatorname{TALL}(\mathrm{John}) \geq \mathrm{d})\}=\{\mathrm{d}: \mathrm{d} \leq 6 \mathrm{ft}\}=(0,6 \mathrm{ft}]$
(74) For any individual $x$, the set of degrees to which $x$ is tall is an initial segment of the height scale, i.e., an interval stretching from the bottom of the scale to a certain point. It is open at the bottom and closed at the top.
(76) Refinement: treat restriction to a certain sort of degrees as a presupposition. amended entry for tall:
$\llbracket \operatorname{tall} \rrbracket=\square \mathrm{d}: \mathrm{d} \square \mathrm{SD} . \square \mathrm{x} . \operatorname{TALL}(\mathrm{x}) \geq \mathrm{d}$
where SD is the set of spatial distances (a proper subset of $D_{d}$ )
i.e., $\llbracket t a l l \rrbracket$ is undefined (rather than false) for degrees that are not heights.
(77) Semantics for the comparative operator:
-er denotes the proper subset relation between sets of degrees:
$\llbracket-\mathrm{er} \rrbracket=\square \mathrm{A} . \square \mathrm{B} \cdot \mathrm{A} \square \mathrm{B}$
(78) Relation to previous semantics in terms of maximality:

If $A$ and $B$ are initial segments of the height-scale and each have a maximal element, then $\mathrm{A} \square \mathrm{B}$ iff $\max (\mathrm{A})<\max (\mathrm{B})$.

Application to an example:
(79) John is taller than Mary.
(80) LF representation:
[-er than [wh ${ }_{1}$ Mary is $t_{1}$ tall] $]_{2}$ [John is $t_{2}$ tall]
(81) interpretation of the two $\square$-abstracts:
$\llbracket \mathbf{w h}_{\mathbf{1}}$ Mary is $\mathbf{t}_{\mathbf{1}} \mathbf{t a l l} \rrbracket=\{\mathrm{d}: \operatorname{TALL}($ Mary $) \geq \mathrm{d}\}$
[ $\mathbf{2} \mathbf{J o h n}$ is $\mathbf{t}_{\mathbf{2}} \mathbf{t a l l} \|=\{\mathrm{d}: \operatorname{TALL}(\mathrm{John}) \geq \mathrm{d}\}$

Applying the meaning of eer:
Truth-condition for (80): $\quad\{\mathrm{d}: \operatorname{TALL}($ Mary $) \geq \mathrm{d}\} \square\{\mathrm{d}: \operatorname{TALL}($ John $) \geq \mathrm{d}\}$ equivalently:
the maximal degree to which John is tall $>$ the maximal degree to which Mary is tall
5.2.2. Semantics of little: DegP-negation
(83) little takes a degree-argument and forms a generalized quantifier over degrees.
(type <d, <dt,t>>)
$\llbracket$ little $\rrbracket=\square \mathrm{d} . \square \mathrm{P}_{<\mathrm{d}, \mathrm{t}} . \mathrm{P}(\mathrm{d})=0$.
intuitively: 'd-little' means 'not d' or 'not to degree d'
(cf. 'not John': $\square \mathrm{P}_{<\mathrm{e}, \mathrm{t}>} \mathrm{P}(\mathrm{j})=0$ )
(85) LF for 'Mary is less tall than John':
$\left[\text {-er than } \mathbf{w h}_{1}\left[\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{3} \mathrm{~J} \text { is } \mathbf{t}_{\mathbf{3}} \text { tall }\right]\right]_{2}\left[\left[\mathbf{t}_{\mathbf{2}} \text { little }\right]_{4} \mathbf{M}\right.$ is $\mathbf{t}_{\mathbf{4}}$ tall $]$
The comparative quantifier is [-er than CP]. little is left behind inside the clause over which the comparative quantifier takes scope.

Because little is left behind in the matrix clause, we expect a matching little in the elliptical 'than'-clause.
t little itself is also a degree-quantifier, so this too needs to take scope at an interpretable position. (The required movement may be very short - just enough for interpretability. In the example, I have moved to the edge of the minimal clause, but there may be lower possible landing sites.)
(86) Interpretation:
$\llbracket \mathbf{3} \mathbf{J o h n}$ is $\mathbf{t}_{\mathbf{3}} \mathbf{t a l l}_{\rrbracket}=\{\mathrm{d}: \operatorname{TALL}(\mathrm{j}) \geq \mathrm{d}\}$
$\llbracket \mathbf{t}_{1}$ little $\|^{\mathrm{g}}=\square \mathrm{P}_{<\mathrm{d}, \mathrm{t}} \cdot \mathrm{P}(\mathrm{g}(1))=0$
$\llbracket \mathbf{t}_{\mathbf{1}}$ little $\mathbf{3} \mathbf{J o h n}$ is $\mathbf{t}_{\mathbf{3}} \mathbf{t a l l} \rrbracket^{\mathrm{g}}=1$ iff $\neg \operatorname{TALL}(\mathrm{j}) \geq \mathrm{g}(1)$
$\llbracket \mathrm{wh}_{\mathbf{1}}\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{\mathbf{3}} \mathbf{J o h n}$ is $\left.\mathbf{t}_{\mathbf{3}} \mathbf{t a l l} \rrbracket=\{\mathrm{d}: \neg \operatorname{TALL}(\mathrm{j}) \geq \mathrm{d})\right\}$
by analogous calculation:
[ $\mathbf{2}$ [ $\mathbf{t}_{\mathbf{2}}$ little] $]_{4}$ Mary is $\mathbf{t}_{\mathbf{4}}$ tall $\rrbracket=\{\mathrm{d}: \neg \operatorname{TALL}(\mathrm{m}) \geq \mathrm{d}\}$
using entry for -er, the complete structure is true iff
$\{\mathrm{d}: \neg \operatorname{TALL}(\mathrm{j}) \geq \mathrm{d}\} \square\{\mathrm{d}: \neg \operatorname{TALL}(\mathrm{m}) \geq \mathrm{d}\}$
equivalently, $\{\mathrm{d}: \operatorname{TALL}(\mathrm{m}) \geq \mathrm{d}\} \square\{\mathrm{d}: \operatorname{TALL}(\mathrm{j}) \geq \mathrm{d}\}$,
i.e., John is taller than Mary.
set-theoretic law which is responsible for this last equivalence:
For any sets A, B such that $A \square U$ and $B \square U$ :
A $\square$ B iff $U-B$U-A.

### 5.2.3. Antonymy

(88) tall and short: ${ }^{1}$
(a) $[[$ tall $]]=\square \mathrm{d}: \mathrm{d} \square$ SD. $\square \mathrm{x} . \operatorname{TALL}(\mathrm{x}) \geq \mathrm{d} \quad$ (see above)
(b) $\quad[$ short $]]=\square \mathrm{d}: \mathrm{d} \square$ SD. $\square \mathrm{x} . \neg \operatorname{TALL}(\mathrm{x}) \geq \mathrm{d}$

This means that short is indeed equivalent to little tall, as Rullmann suggested:
(89) (a) John is $\mathbf{t}_{\mathbf{1}}$ short
$\neg$ TALL (John) $\geq \mathrm{g}(\mathbf{1})$
(b) John is $\left[\mathbf{t}_{\mathbf{1}}\right.$ little] tall

## LF: $\quad\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{\mathbf{2}}$ John is $\mathbf{t}_{\mathbf{2}}$ tall

$[\square \mathrm{P} . \mathrm{P}(\mathrm{g}(\mathbf{1}))](\square \mathrm{d} . \operatorname{TALL}(\mathrm{John}) \geq \mathrm{d})$
equivalent to: $\neg$ TALL $(J o h n) \geq g(\mathbf{1})$
Confirmation for this equivalence:
(90) (a) John is taller than Mary.
(b) Mary is shorter than John.
(c) Mary is less tall than John.

All three of these are equivalent.
But little tall differs from short in containing a separate, movable, constituent. Therefore, the two will not always be equivalent in more complex sentences: little, as we will see, can sometimes take non-local scope, giving rise to readings not shared by short.

## 6. Ambiguity of less as a scope ambiguity

The covert little in the than-clause can take scope either below or above the modal operator allowed. This will give us the two readings:
(91) little above allowed: below-maximum reading
little below allowed: below-minimum reading

[^0](92) Lucinda drove less fast than she was allowed to.
(93) LF-(i): [-er than $\mathbf{w h}_{\mathbf{1}}\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{3}$ was allowed $L$ to drive $\mathbf{t}_{\mathbf{3}}$ fast $]_{2}$
[ $\mathbf{t}_{\mathbf{2}}$ little] $\mathbf{4}_{\mathbf{L}} \mathrm{L}$ drove $\mathbf{t}_{\mathbf{4}}$ fast
interpretation:
$\{\mathrm{d}: \neg \square \mathrm{w} \square$ Acc: L drives d-fast in w$\} \square\{\mathrm{d}: \neg \mathrm{L}$ drives d-fast $\}$ equivalently:
$\{\mathrm{d}$ : L drives d-fast $\} \square\{\mathrm{d}: \square \mathrm{w} \square$ Acc: L drives d-fast in w $\}$
i.e., her actual speed is below her highest allowable speed

LF-(ii): [-er than wh $\mathbf{1}_{\mathbf{1}}$ was allowed $\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{\mathbf{3}} L$ to drive $\mathbf{t}_{\mathbf{3}}$ fast $]_{\mathbf{2}}$
[ $\mathbf{t}_{\mathbf{2}}$ little] $\mathbf{4}_{\mathbf{4}} \mathrm{L}$ drove $\mathbf{t}_{\mathbf{4}}$ fast
interpretation:
$\{\mathrm{d}: \square \mathrm{w} \square$ Acc: $\neg \mathrm{L}$ drives d-fast in w$\} \square\{\mathrm{d}: \neg \mathrm{L}$ drives d-fast $\}$ equivalently:
$\{\mathrm{d}: \neg \square \mathrm{w} \square$ Acc: L drives d-fast in w\}\{d: $\neg \mathrm{L}$ drives d-fast $\}$
$\{\mathrm{d}$ : L drives d-fast $\} \square\{\mathrm{d}: \square \mathrm{w} \square$ Acc: L drives d-fast in w$\}$
i.e., her actual speed is below her lowest allowable speed

(96) I am amazed that we are allowed to teach so little.
ad hoc semantics for I am amazed that .... so .....:
\{d: .... d .... \} is larger than I expected.
(98) LF-(i): little above allowed:
[so ${ }_{1}$ little] $]_{3}$ are [allowed [we to teach $\mathrm{t}_{3}$ much]] interpretation:
$\{\mathrm{d}: \neg \square \mathrm{w} \square$ Acc: we teach d-much in w$\}$ is larger than I expected reading: I expected a higher maximum (i.e., a permission to teach more)

LF-(i): little below allowed:
are allowed [[s $\mathrm{so}_{1}$ little] $]_{3}\left[\right.$ we to teach $\mathrm{t}_{3}$ much]] interpretation:
$\{\mathrm{d}: \square \mathrm{w} \square$ Acc: $\neg \mathrm{we}$ teach d-much in w$\}$ is larger than I expected reading: I expected a higher minimum (i.e., a requirement to teach more)

Summary: desirable features of the new analysis

- new features (advantages over both of Rullmann's analyses):
(100) There are no ambiguous morphemes and no morphemes that only are interpretable as part of idioms. -er, little each have meanings of their own, and each are unambiguous. The observed ambiguity is structural (a scope ambiguity). The scoping mechanism that creates it is independently needed (for interpretability).
(101) The account of ambiguity in less-comparatives generalizes to ambiguous constructions with how little, so little.
- inherited features (preserved from Rullmann's analyses):
(102) Ambiguity is not shared by more-comparatives. Reason: The extra piece, little, whose variable scope matters, is not present there. (Maybe an extra much is there, but if so, it is semantically vacuous or trivial in such a way that its scope doesn't affect meaning.)
(103) Ambiguity disappears when the than-clause is too simple. Reason: no room for nontrivial scope differences.
(104) Ambiguity disappears when mimimum or maximum doesn't exist (same pragmatic explanation as before).

Some prima facie disadvantages and unexplored consequences

- explanation of negative island effect:

Rullmann, following v. Stechow, attributes the deviance of (105) to the fact that the set of degrees to which Mary isn't tall has no maximum:
(105) *John is taller than Mary isn't.

But the present analysis doesn't refer to maxima. (105) is predicted to have a well-defined meaning:
$\{\mathrm{d}: \neg$ TALL $($ Mary $) \geq \mathrm{d}\} \square\{\mathrm{d}: \operatorname{TALL}($ John $) \geq \mathrm{d}\}$
Tentative suggestion: (105) is deviant because (106) is necessarily false. A final segment of the scale can never be a proper subset of an initial segment. The same problem arises with (107). ${ }^{2}$
(107) *John is taller than Mary is short.

But there are still problems. (108) should be fine, in fact the same as (109).
(108) *John is shorter than Mary isn't tall.
(109) John is shorter than Mary is.

Maybe we need a syntactic account of the negative island effect, after all? (See Beck 1996, Beck \& Rullmann 1999 for more discussion.)

- predictions about NPI licensing:
(110) [-er $\left.\square_{\text {than-clause }}\right] \square_{\text {matrix-clause }}$ the set of degrees satisfying $\square \square$ the set of degrees satisfying $\square$

Here $\square$ is a downward entailing context.
Reason: if $\mathrm{A}^{\prime} \square \mathrm{A}$, then $\mathrm{A} \square \mathrm{B}$ implies $\mathrm{A}^{\prime} \square \mathrm{B}$.
correct prediction: NPIs are okay in the (immediate) scope of -er within the 'than'clause.
(111) structure of less-comparatives:

$$
\left[\text {-er }\left[\text { than-clause }=\square \cdots . \operatorname{little}\left[_{\text {scope-of-little }}=\square \cdots\right]\right] \square_{\text {matrix-clause }}\right.
$$

[^1](112) Recall from above (see e.g. (93), (94)):
less-than-maximum reading arises when little is "high" less-than-minimum reading arises when little is "low"
(113) Recall from Rullmann:
less-than-maximum reading does not license NPIs less-than-minimum reading does license NPIs
(114) correct prediction: NPIs in between -er and little should be okay.

Reason: this is a DE context, as shown in (110).
Thus, if little is low and NPI is above it, that's okay.
(115) potential problem: what about when little is high and NPI is inside the scope of little in the 'than'-clause?

If entailment patterns are tested for the whole structure in (111), then we find that $\square$ is not a DE context. This would give the right prediction, viz. that clauses with high little have no room for NPIs.
(116) meaning of $(111)^{3}$ : the complement of the set of degrees satisfying $\square \square$ the set of degrees satisfying $\square$ If $A^{\prime} \square A$, then $D_{d}-A \square B$ does not imply $D_{d}-A^{\prime} \square B$ (rather, the reverse, so $\square$ is a UE context).

But if we just look at the local structure containing $\square$ in (111), NPIs should be licensed there, since they are in the scope of little, which is a negation.

Speculation: -er and high little "cancel each other out", i.e., act as a unit for NPI-licensing. Does this happen elsewhere?
(117) He didn't say that nobody ate anything.
(118) ?He didn't feed nobody anything.
(119) ??No fewer than five people ate anything.

- intermediate readings:

Unlike Rullmann's version, which always predicts (at most) two readings for each occurrence of
${ }^{3}$ This is assuming that there is nothing between -er and little. The monotonicity result obtains more generally as long as there is nothing non-UE in that region.
little, the present analysis in principle allows intermediate scopes for little when there is more than one other scope-taking element in the 'than'-clause. Is this option ever attested?

## 6. Further predictions: necessity; modality in matrix

The ambiguous examples treated by Rullmann all involve possibility operators in the thanclause. Our analysis predicts analogous ambiguities when (a) the than-clause contains a necessity operator, or (b) there is a modal operator in the matrix. These predictions are borne out for both cases.

Necessity in 'than'-clause
(120) It was easy to get an A in this class. You only had to do $60 \%$ of the assignments. Still, John only got a B.
He turned in less than was required.
'less-than-minimum' reading
(121) To get an A in this course, you had to solve all these problems in 45 minutes or less. John had no problem.
He took even less time than was required.
'less than maximum' reading
(122) little in than-clause scopes over required:
than $\mathbf{w h}_{\mathbf{1}}\left[\mathbf{t}_{\mathbf{1}} \text { little }\right]_{\mathbf{2}}$ required [to turn in $\mathbf{t}_{\mathbf{2}}$ much]
$\{\mathrm{d}: \neg \square \mathrm{w} \square$ Acc: John turns in d-much in w$\} \square\{\mathrm{d}: \neg$ John turn in d-much $\}$
$=$ 'less than minimum' reading
(123) little in than-clause scopes below required:
than $\mathbf{w h}_{\mathbf{1}}$ required $\left[\mathbf{t}_{\mathbf{1}} \text { little] }\right]_{\mathbf{2}}$ [to take $\mathbf{t}_{\mathbf{2}}$ much time]
$\{\mathrm{d}: \square \mathrm{w} \square$ Acc: $\neg$ John takes d-much time in w\} $\square\{\mathrm{d}: \neg$ John took d-much time $\}$
$=$ 'less than maximum' reading
Modal operator (necessity or possibility) in matrix clause
(124) We are allowed less than 7 hours of sleep.
'it is not allowed that we sleep (as much as) 7 hours'
(125) We are allowed to have less than 7 hours of prayer.
'it is allowed that we don't pray (as much as) 7 hours'
(126) We are allowed to have less than 7 hours of sleep. [judgment?]
(127) We are allowed less than 7 hours of prayer. [judgment?]
(128) I need less than 5 hours of sleep (to stay healthy). 'it is not necessary that I have (as much as) 5 hours of sleep'
(129) I need to have less than 5 hours of prayer (to maintain my sanity). 'it is necessary that I not have (as much as) 5 hours of prayer'
(130) I need to have less than 5 hours of sleep (to stay healthy). [judgment?]
(131) I need less than 5 hours of prayer (to maintain my sanity). [judgment?]

A minimal contrast between less fast and more slowly:
(132) John and Mary want to be in Boston by 8 pm (at the latest). Currently, John is 30 miles away from Boston, and Mary is 40 miles away from Boston. Therefore:
(a) She needs to drive faster than he needs to drive.
(b) He needs to drive less fast than she needs to drive.
(c) \# He needs to drive more slowly than she needs to drive.


[^0]:    ${ }^{1}$ This is (almost) the treatment of antonymy (polar opposition) in Stechow (1984b) and Kennedy (1997, 2000). See also Bierwisch (1987) and Meier (2000).

[^1]:    ${ }^{2}$ See Kennedy on "cross-polar anomaly".

