3.032 Problem Set 1
Fall 2006
Due: Start of lecture, 9/15/06

1. Thin-film silicon nitride cantilevers such as the one shown in Figure 1 are used in scanning probe microscopy, resonant frequency measurements, and electrostatic actuation. Let us approximate this cantilever as a clamped-free beam with a length of 200 μm, a width of 30 μm, and a thickness of 0.8 μm. Take the Young's modulus of thin-film silicon nitride to be 210 GN/m^2. The weight of the cantilever can be neglected.

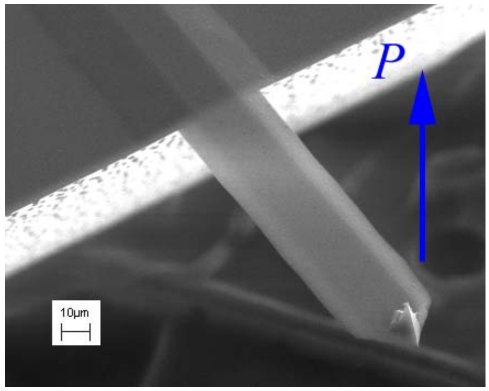

Image by OpenCourseWare. Adapted from work by KristianMolhave, Wikimedia Commons.

Figure 1: Thin-film silicon nitride cantilever.

   (a) If the free end of the beam contacts a surface (represented by a point load in Figure 1), and is deflected upwards by 1.0 μm, find the magnitude of the point load.

   (b) Draw a free-body diagram of the beam. Sketch the shear and bending moment in the beam along its length, labeling maximum and minimum values. Use the following sign convention for positive shear and bending moment [Beer and Johnson, *Mechanics of Materials* (1992)]:

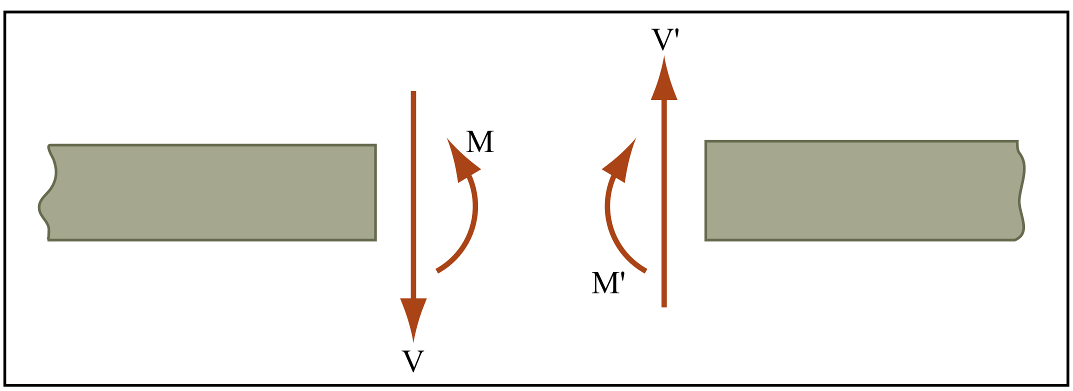

Figure by MIT OpenCourseWare.

   (c) In your scanning probe experiments, you need to be able to measure point loads that are only one-tenth of the value you calculated above. Assuming that a deflection of 1 μm produces a suitable signal for detection, how much longer does the cantilever need to be?

2. You are assembling a structure in a heavy wind, which acts as a distributed load q (force per length) on one of the wide boards you are erecting (Figure 2). Your friend

(weight 150 lb) can just barely keep the board upright by hanging from a flexible cord connected to the board. The board is connected to the ground by a pin joint, and the mass of the board is 30 kg/m. Ignore the effect of the wind on your friend.

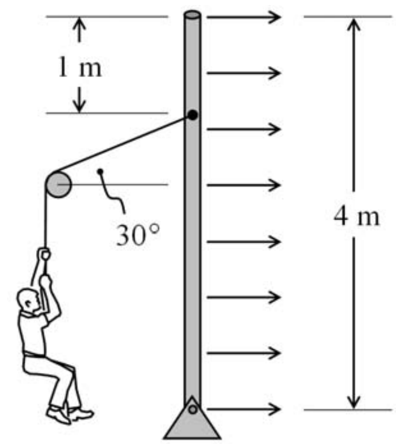

Figure 2: Upright board subject to a distributed load and a point load.

(a) Draw a free-body diagram of the board and find the wind load q and the reaction forces at the pin joint.

(b) Sketch the shear and bending moment in the board.

3. The Harvard Bridge on Mass. Ave. (Figure 3(a,b)) was rebuilt in the late 1980s due to the fact that the pin-and-hanger assemblies of the expansion joints on the bridge were the same as those of the Mianus River Bridge in Connecticut. The Mianus River Bridge collapsed in 1983 when a single pin became overloaded and caused the death of three people and serious injury of three others. Your task is to find the normal and shear forces and moment associated with that failure, which occurred at the position marked **x**, the midspan between pins C and D (Figure 3(c)).

   (a) Draw a free body diagram of the entire pin-and-hanger assembly. The weight of the assembly can be neglected.

   (b) Determine the reaction forces about joint B.

   (c) Determine the forces along and normal to the horizontal beam, and the moment at the point of failure in the steel beam.

4. Nanowires of amorphous silica (SiO_2) (Figure 4) are considered as possible optical waveguides in miniaturized electronics. For handling and assembly purposes, it is necessary to know and be able to measure the critical buckling load P_{cr} of such nanowires.

Image courtesy flickr user afagen.

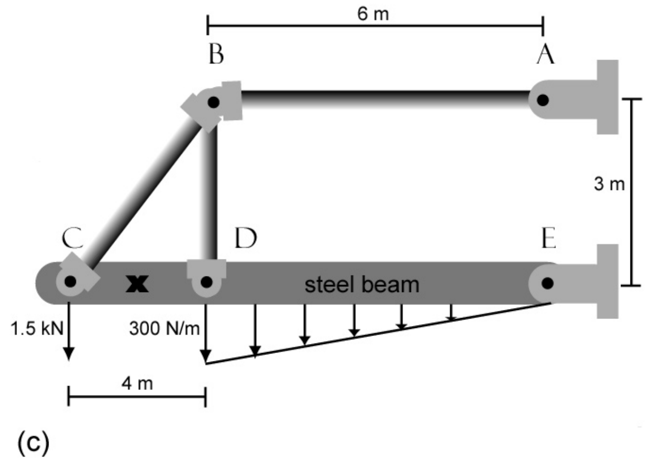

Figure 3: (a,b) Harvard Bridge; (c) Pin-and-hanger assembly, adapted from Hibbeler (2005).

Fig. 4b in Tong, Limin, et al. "Subwavelength-Diameter Silica Wires for Low-Loss Optical Guiding." *Nature* 426 (December 2003): 816-819.

Figure 4: Silica nanowire. (a) Amorphous silica fibers act as optical waveguides [Tong et al., *Nature* 426: 816 (2003)]. (b) Critical buckling load can be measured via compression with instruments such as atomic force microscopes.

(a) For the above nanowire of 400 nm diameter and 1 mm length, determine the critical buckling load. Assume the value of the Young's modulus E for bulk silica, which is $70\,\mathrm{GN/m^2}$.

(b) Using the cantilever in Problem 1, what is the deflection of that cantilever that you would need to achieve to impose this elastic instability?