3.032 Problem Set 7
Fall 2006
Due: 12/08/06 before the start of lecture

1. Creep can occur via several different mechanisms in amorphous and crystalline materials, each with their own activation energy Q. Typically, there is an Arrhenius behavior to this creep response, with an exponential prefactor that is particular to the material considered and on the operating mechanism.

    (a) You are running two separate creep experiments on polycrystalline Ni, one at 527^oC and the other at 532^oC. You observe that $d\varepsilon/dt$ increases by a factor of 1.5 for the sample tested at the higher temperature. What is the activation energy Q for this creep mechanism?

    SOLUTION: Take the ratio of the two strain rates expressed as Arrhenius equations. Since the temperatures differ very little, you would expect the same mechanism and therefore the same Q at both temperatures. Remember to express temperature in [K], since R is in [J/mol K]:

    $$\frac{\epsilon'_1}{\epsilon'_2} = exp(-Q/RT_1 + Q/RT2) \tag{1}$$

    Solving for Q,
    $$Q = \frac{ln(2/3)}{1/RT_2 - 1/RT_1} = 434 kJ/mol \tag{2}$$

    (b) What do you think the mechanism at 532^oC could be, and why? Note that the actual grain size d and stress magnitude need not be stated to figure this out, but you can use deformation mechanism maps to narrow down your options.

    SOLUTION: The melting temperature of Ni is 1455C, or 1728K, so this temp of 805K is at $T/T_m = 0.46$. Referring to deformation mechanism maps of Ni for several grain sizes, the options would depend on the magnitude of stress applied, and could include elastic behavior, Coble creep, or dislocation creep (and, at really high stress, dislocation glide). Since this must be creep (we're measuring a strain rate, so the material is creeping!), our options are Coble or dislocation creep. Have to know the stress to say for sure, but Coble creep will dominate until normalized stresses get to be around 10^{-3}.

    (c) You are running these tests because you would like to develop a new polycrystalline Ni alloy for Boeing aircraft engines. The Ni parts will be under a constant stress of about 1.3 GPa due to centrifugal forces, and a temperature of about 1100^oC. Which creep mechanism are you most concerned about, and how do you propose to modify the polycrystalline Ni alloy to minimize creep rates for this application? *SOLUTION:*

To locate my position on a deformation mechanism map, I note that $T/T_m = 0.79$ and that normalizes stress is about 1.3 GPa/13 GPa = 0.1. All creep mechanisms could operate, but the most concerning mechanism is the dominant one. At this normalized temperature, Coble creep and dislocation creep are likely, though deformation will also occur via dislocation glide. Note that dislocation glide is NOT a diffusive mechanism, but dislocation creep is diffusive (vacancies diffuse to dislocations to enable climb). To inhibit Coble creep, I would increase the grain size d to reduce the volume fraction of grain boundaries comprising the material. To inhibit dislocation creep, I would inhibit diffusion of vacancies (or self diffusion of Ni) by, say, alloying. Note that since yield strength relates to the difficulty of dislocation glide and will decrease as I increase the grain size, this approach would also make the material weaker UNLESS the alloying compensated for the increased grain size's deleterious effect on strength. Tradeoffs.

2. One of your colleagues proposes that the advancing crack in the cast copper-and-tin Liberty Bell in Philadelphia, PA could have been halted early on by drilling a small hole just beyond the crack's termination point. Would this be likely to work? Why or why not?

   SOLUTION: Yes, placing a hole right ahead of the crack tip, effectively blunting the crack by increasing the tip radius, should work. This proposal is discussed by Hertzberg (p. 274), who attributes it to Wilfred Jordan (1915). The approach would likely halt the crack by relieving the sharp crack-tip radius (and therefore the high stress concentration) in a way that the alloy is evidently unable to do on its own through plastic deformation. However, one would need to be sure that the hole is indeed drilled ahead of the crack tip and that the drilling process creates a sufficiently smooth hole that new cracks are not initiated. Additionally, this solution does not address the material's fundamental inability to resist crack propagation in other locations.

3. Earlier this semester, you learned about the stresses generated in thin-wall pressure vessels and how to calculate the minimum necessary wall thickness to avoid yielding. However, yielding that leads to localized deformation and leaking is far more preferable than the alternate failure mode of rapid crack propagation.

   (a) Assuming Mode I failure, obtain an expression for the maximum crack length in a vessel wall that would allow yielding before crack propagation, assuming a factor of safety of two.

   (b) Select four materials (any class) and rank them by their ability to satisfy the yield-before-break criterion for this application, referencing your data sources. Clearly state any assumptions.

   SOLUTION: The allowable stress is $\frac{\sigma_y}{2}$ where σ_y is the yield stress. Applying this value to the relationship between mode I fracture toughness K_{Ic}, crack length a, and stress, we have

   $$K_{Ic} = \frac{\sigma_y}{2} f \sqrt{\pi a}$$

$$a_c = \frac{4}{\pi f^2} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$$

where a_c is the critical crack length and f is a geometrical factor.

(Note that the stress is related to the pressure p in a cylindrical pressure vessel as

$$\sigma = \frac{\sigma_y}{2} = \frac{pr}{t}$$

where r and t are the radius and thickness of the vessel, respectively.)

The figure of merit (number that allows you to compare outcomes on an equal basis) for ranking different materials is $\left(\frac{K_{Ic}}{\sigma_y} \right)^2$ or, equivalently for material ranking, $\frac{K_{Ic}}{\sigma_y}$. To achieve the yield-before-break criterion, we are interested in materials with a high K_{Ic} and a low σ_y, even though a low σ_y limits the application pressure. (In other words, designing the tank for high pressures isn't worth it if one day the tank explodes without warning).

4. A plate of glass is placed under a tensile stress of 60 MPa.

   (a) Determine the maximum surface flaw size that can be accommodated without fracture. Take the specific surface energy of the glass to be $0.3\,\text{J/m}^2$.

   (b) What instruments could you use to visualize flaws of that size, in order to rule in/out the likelihood of such flaws in the as-processed glass?

   (c) Aluminum has the same Young's elastic modulus as glass. Compare the maximum flaw size that could be tolerated in Al without fracture, clearly justifying any assumptions.

   SOLUTION: Let us assume a condition of plane stress. The critical crack length a_c is (M&C equation 7.6a)

   $$a_c = \frac{2E\gamma}{\pi\sigma^2} \approx 3.7\,\mu\text{m}$$

   where the elastic modulus is taken to be 70 GPa for glass.

   This size crack is large enough be detected optically. Alternatively, scanning electron microscopy could be used, but this is challenging on materials like glass that are not conductive; you'd have to coat it with gold like you did in lab with the rat tail tendon, and that could obscure the crack length.

   Fracture in aluminum involves a significant amount of plastic deformation, so we would not want to simply recalculate a critical crack length using the surface energy of aluminum. Alternatively, we might use the K_{Ic} equation with $K_{Ic} \approx 30\,\text{MPa}\,\sqrt{\text{m}}$ (aluminum alloys typically have a fracture toughness in the tens of MPa $\sqrt{\text{m}}$). The critical crack length a_c is thus estimated to be about 8 mm—far larger than the corresponding crack length for glass!

5. One of the ways that the yield strength and toughness of alloys like steel are controlled is through a process called tempering, which means heating the material well below the melting temperature to relieve internal stresses via defect annihilation. Unfortunately, strength as

measured by σ_y is generally reduced via tempering, whereas toughness as measured by K_{Ic} is increased via tempering.

For example, AISI 4340 steel can go from (σ_y = 1190 MPa; K_{Ic} = 135 MPa $\sqrt{m}$) to (σ_y = 1620 MPa; K_{Ic} = 60 MPa $\sqrt{m}$), varying almost linearly through this range of tempering times/temperatures. It is often important to figure out whether a ductile material component like steel will fail by yielding or by fracture in a particular application such as a vertical member in tension.

(a) Given that visual inspection of the 4340 steel member can detect all cracks greater than a = 2 mm, determine whether the toughest steel noted above will fail by yielding or fail by fracture. Assume a through-thickness center crack in a semi-infinite body, such that f = 1.

SOLUTION:

$$K_{Ic} = (f = 1)\sigma \sqrt{\pi a} - - > \sigma_f = \frac{135 MPa \sqrt{m}}{\sqrt{\pi * 2x10^{-3}m}} = 1.7 GPa \qquad (3)$$

Since this failure stress exceeds the yield stress, the material will pass through yielding before this stress is attained (fail by yielding).

(b) Ideally, the steel should carry the maximum possible load without yielding or fracture. That means it should be processed so that the yield and fracture strengths are the same value. Calculate $\sigma_f(K_{Ic})$ over the range of $50 \leq K_{Ic}$ 150 MPa $\sqrt{m}$. Plot this result on a graph of stress (y-axis) vs. K_{Ic} (x-axis), identifying this ideal yield/fracture strength value as the stress at which $\sigma_y(K_{Ic})$ and $\sigma_f(K_{Ic})$ cross on this graph.

SOLUTION: Two straight lines are graphed. The first is given by the (K_{Ic}, σ_y) data given in the problem statement, and has a negative slope. The other are the failure stresses as a function of K_{Ic}, as shown in (a), and has a positive slope. These lines cross at a stress of about 1400 MPa.

6. Review the paper, "Fatigue behavior of nanocrystalline metals and alloys", Hanlon et al., Int. J. Fat. 27 (2005).

(a) In your own words, explain why the reduction of grain size d is generally considered a desirable microstructure to increase fracture and fatigue resistance in metals.

SOLUTION: Reduced grain size generally increases yield strength, with one explanation of this stated as "Small grains leave little room for dislocation pileup between closely space grain boundaries, so the externally applied stress must be increased to induce general plastic flow in many grains." This is true, but is not an answer for fracture and fatigue resistance. Fracture resistance implies that a material can dissipate strain energy by means other than fracture (e.g., plasticity). Materials with high yield strength also exhibit high hardness. This means it takes alot of energy to rearrange

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bonds, and even more to break them. If it takes alot of energy to nucleate a crack, the material is fracture resistant. If it takes a lot of energy to nucleate a crack, it takes even more to grow that crack enough to generate fatigue failure (a combination of plastic deformation and crack propagation under cyclic loading), so the material is resistant to fatigue failure.

(b) In Section 2.2, the authors detail the experimental conditions for this fatigue life study. Were the fatigued samples ever loaded in compression? How do you know?

SOLUTION: R was always greater than zero, so $\sigma_{min}/\sigma_{max}$ ¿ 0. This implies tension.

(c) The authors first computed the stress intensity factor K_I according to Eq. 1. Given that they observed later that the range of a/W they accessed was 0.1 to 0.7, what was the range of K_I that they explored, according to Eq. 1?

SOLUTION: The range of $f(a/W)$ can be computed from this to range 2.1 to 10.6, so K_{min} and K_{max} ranged from $2.1\sigma\sqrt{a}$ to $10.6\sigma\sqrt{a}$. Since you know w, you find the range of a and also know the stress σ applied, so you can compute actual [MPa $\sqrt{m}$].

(d) What is the *J*-integral, why did the authors (and anyone else) use it to consider this material's fracture behavior, and what was the range of the *J*-integral they explored, according to Eqs. 1 - 2?

SOLUTION: The J-integral is the energy required to separate crack faces, and provides a way to calculate valid K_{Ic} for small samples in which the plastic zone size r_p might not be small compared to the crack and sample dimensions. The range of J can be calculated from Eq. 2 for the range of f(a/w) explored (0.1 to 0.7), but can also be calculated directly from its relationship to K_{Ic}.

(e) In Section 3, the authors relate their results. Why do you think the ductility of the nanocrystalline, ultrafine crystalline, and microcrystalline Ni ranked the way they did?

SOLUTION: The authors found nanocrystalline samples to be the least ductile because slip of dislocations from one grain to the next was inhibited. The stress had to be applied (by the machine) to force dislocations into new grains, since pileup of dislocations to create a driving force for slip into the next grain was not possible (no room for pileup). When dislocation plasticity is no longer an easy option, energy is dissipated by other means such as fracture. Thus, the material fractures rather than yields at small strains for small grain sizes.

(f) What is the fatigue limit (i.e., stress amplitude in [Pa]) of nc, ufc, and mc Ni, according to Hanlon et al., Scripta Mater. (2003) as reproduced in Fig. 4?

SOLUTION: The stress range at which the fatigue life was independent of actual stress

magnitude was 200 MPa (mc), 350 MPa (ufc) and 400 MPa (nc), so it increased as grain size decreased.

(g) What is the fatigue crack growth threshold or threshold stress intensity factor ΔK_o for nc Ni at the various load ratios considered, according to Fig. 5?

SOLUTION: The ΔK_o is the value at which measurable crack growth starts: 4 MPa $\sqrt{m}$ (R = 0.1), 3 MPa $\sqrt{m}$ (R = 0.3), and 2.5 MPa $\sqrt{m}$ (R = 0.7).

(h) Assuming that the cyclic crack growth characteristics of nc and ufc Ni are well characterized by the Paris law, determine the values of *C* and *m* for both materials from the data in Figs. 5a-c. From this analysis, state whether you find the fatigue fracture lifetime to be a stronger function of microstructure or of loading conditions, and why you think this to be the case.

SOLUTION: This requires fitting lines to these data shown as da/dN vs. ΔK on log scales. Easiest way is to take the ratio of two points to solve for m, and then insert that m back into one such equation to solve for C. Alternatively, one can use ΔK_o and the value of da/dN at this threshold to insert into the Paris law equation:

$$log(da/dN) = m * log(\Delta K) - x \tag{4}$$

where x = the power of 10 related to C. I prefer the former approach. For example, for R = 0.3: for nc, m = 3.5 and C = 10^{-8}; for ufc, m = 4.0 and C = 10^{-10}. Comparing values obtained this way for all R, I find that the C and m varied more between the materials (e.g., m ~ 3.5 for nc, but 4.0 for ufc) than as a function of R for a given material. This means the fatigue response is a strong function of the microstructure.

(i) Briefly explain the mechanisms you understand to be responsible (either from the paper or your own knowledge via 3.032 and other sources) for the chief conclusion of the authors: Grain size reduction correlated with decreased subcritical fatigue fracture in nc Ni, but also to an increased total fatigue life.

SOLUTION: As grain size decreases, the material gets harder (more resistant to plastic deformation), and therefore more resistant to nucleation of new cracks (energy dissipated by plasticity if possible, then fracture). However, once a crack is nucleated, small grain size enables a straighter crack path: the crack does not need to make large turns as it works its way around grains (assuming transgranular fracture for simplicity). If it's a straight crack, it will feel that Mode I loading at full strength, and continue to open under that tensile stress to grow quickly. The overall fatigue life increases due to the resistance to crack nucleation and subcritical crack growth, not to a more favorable da/dN.

7. Select one topic that has been covered this semester in 3.032 (other than visco/rubber elasticity) and contribute to the corresponding Wikipedia article (or create a new article if none

exists). This is a good opportunity to get up to speed in any areas in which you lack confidence.

- An ideal contribution will be several paragraphs of correct and useful information, possibly including equations or figures.
- Make sure to reference your sources correctly.
- It will make grading easier if you make a single update (proofread your work first) rather than multiple updates. Please print out your contribution *or* indicate the article URL and your user name in your pset.

SOLUTION: Excellent job on topics ranging from mechanisms of plasticity to to Coble creep to nanoindentation to ductile fracture. Well done!

Congratulations on completion of your last graded pset for 3.032!