Lecture 4 Honeycombs Notes, 3.054

Honeycombs-In-plane behavior

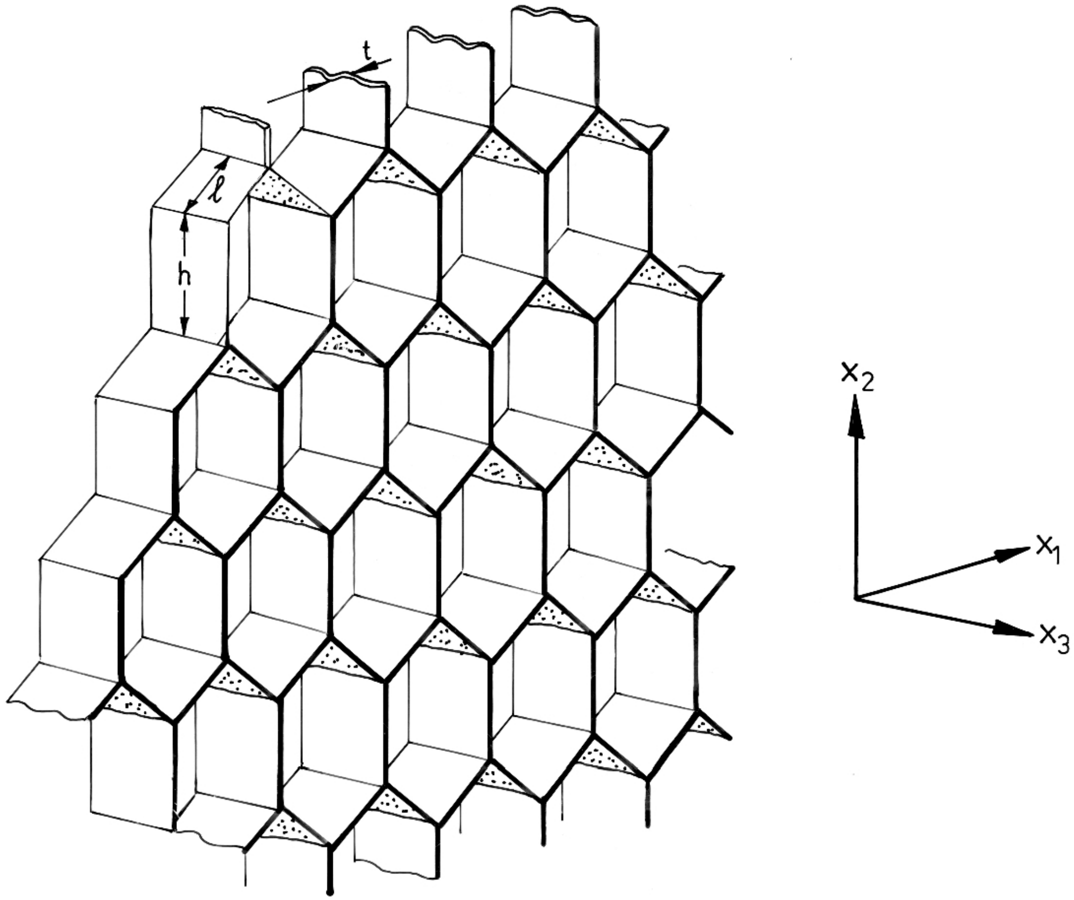
- Prismatic cells
- Polymer, metal, ceramic honeycombs widely available
- Used for sandwich structure cores, energy absorption, carriers for catalysts
- Some natural materials (e.g. wood, cork) can be idealized as honeycombs
- Mechanisms of deformation and failure in hexagonal honeycombs parallel those in foams
  - simpler geometry — unit cell — easier to analyze
- Mechanisms of deformation in triangular honeycombs parallel those in 3D trusses (lattice materials)

Stress-strain curves and Deformation behavior: In-Plane

Compression

- 3 regimes
  - linear elastic – bending
  - stress plateau
    - buckling
    - yielding
    - brittle crushing
  - densification – cell walls touch
- Increasing $t/l \Rightarrow E^* \uparrow \quad \sigma^* \uparrow \quad \epsilon_D \downarrow$

Honeycomb Geometry



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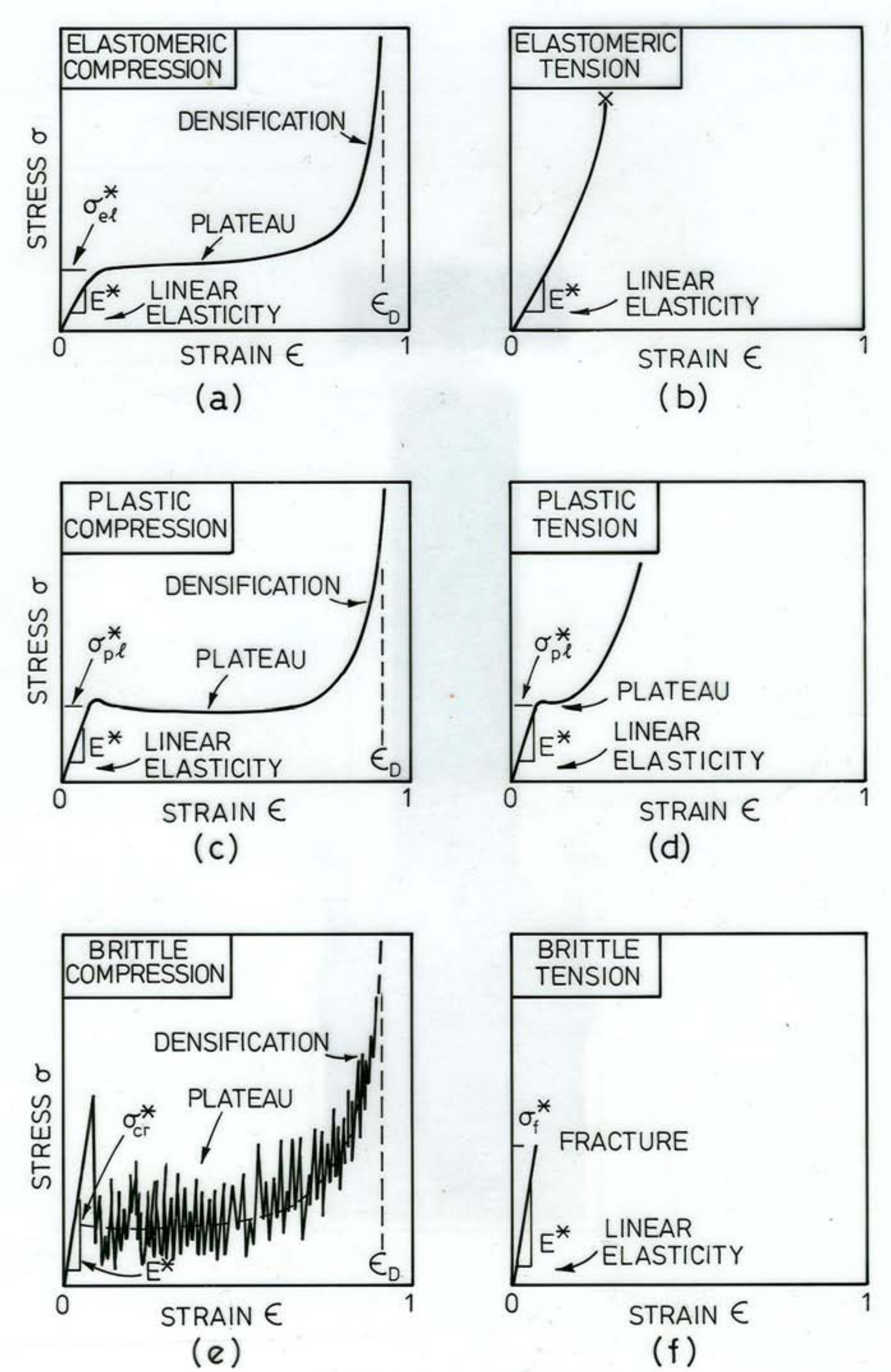

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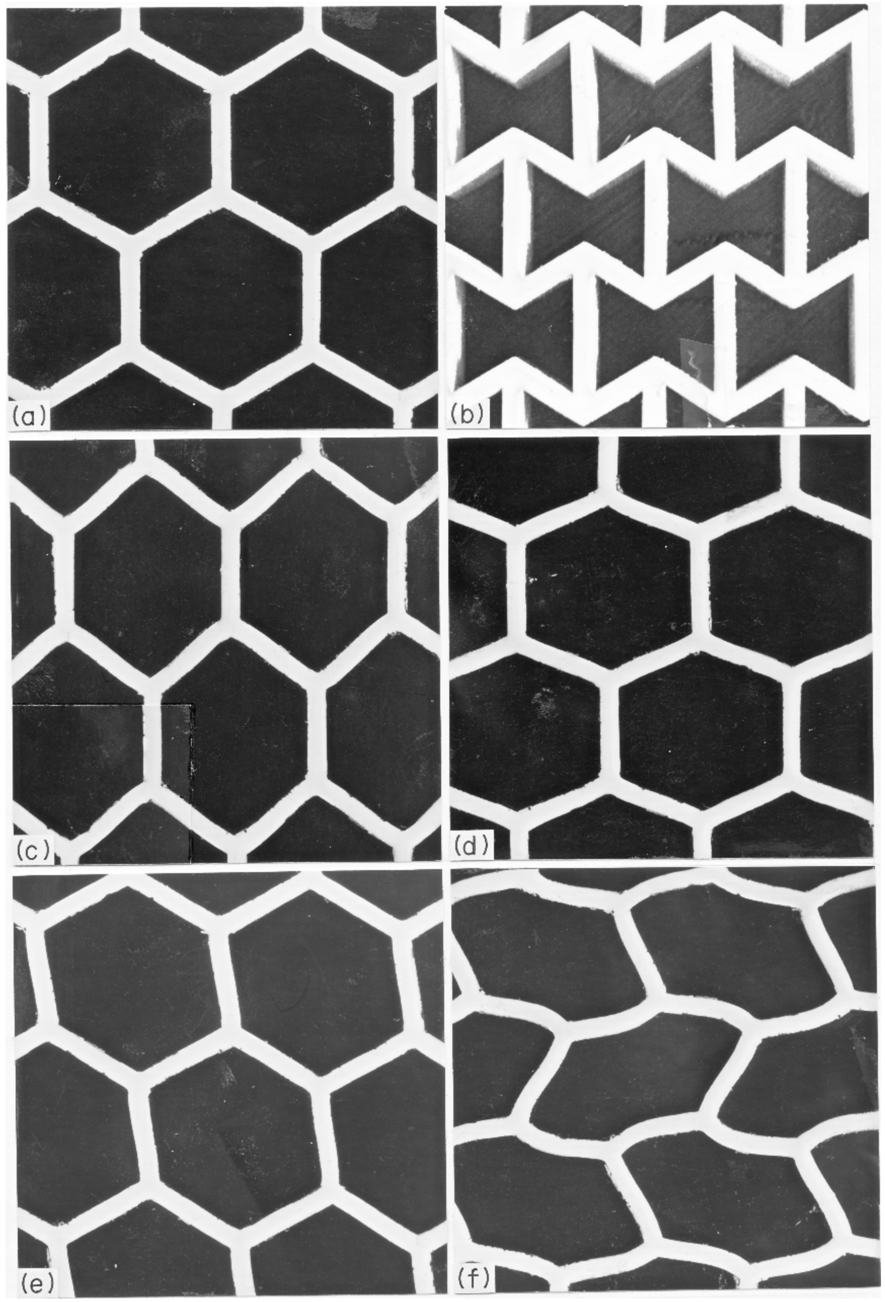
Deformation mechanisms

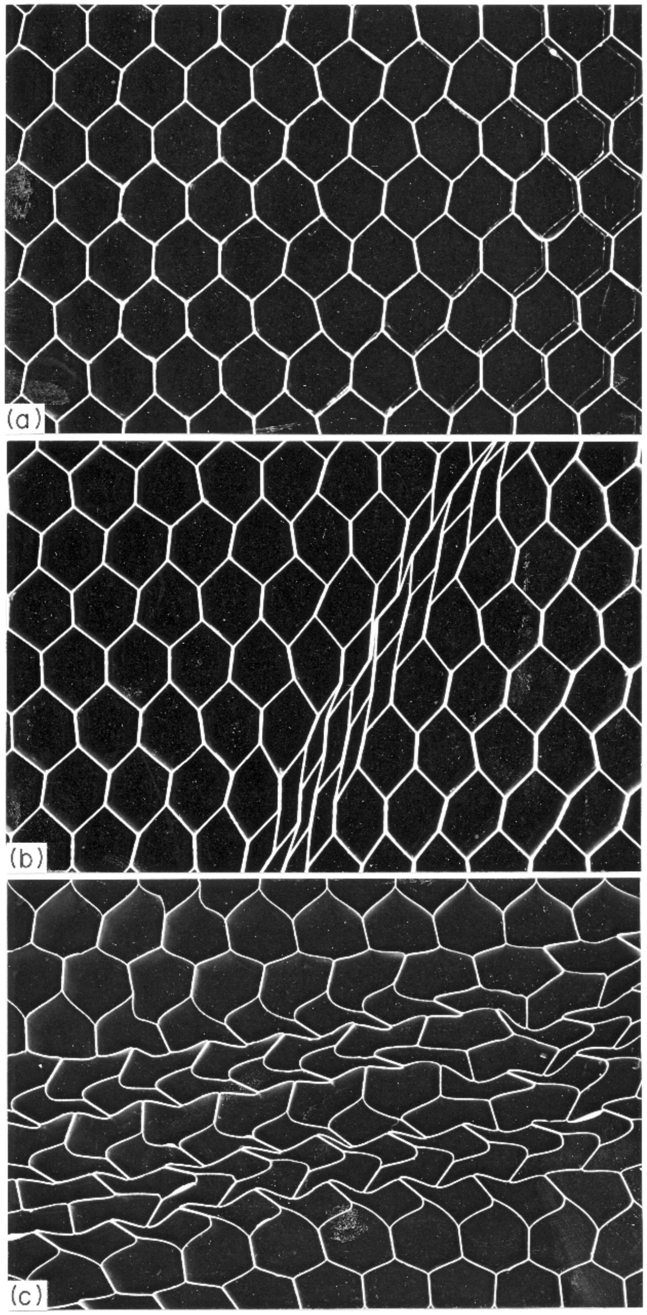
Bending
X_1 Loading

Bending
X_2 Loading

Bending
Shear

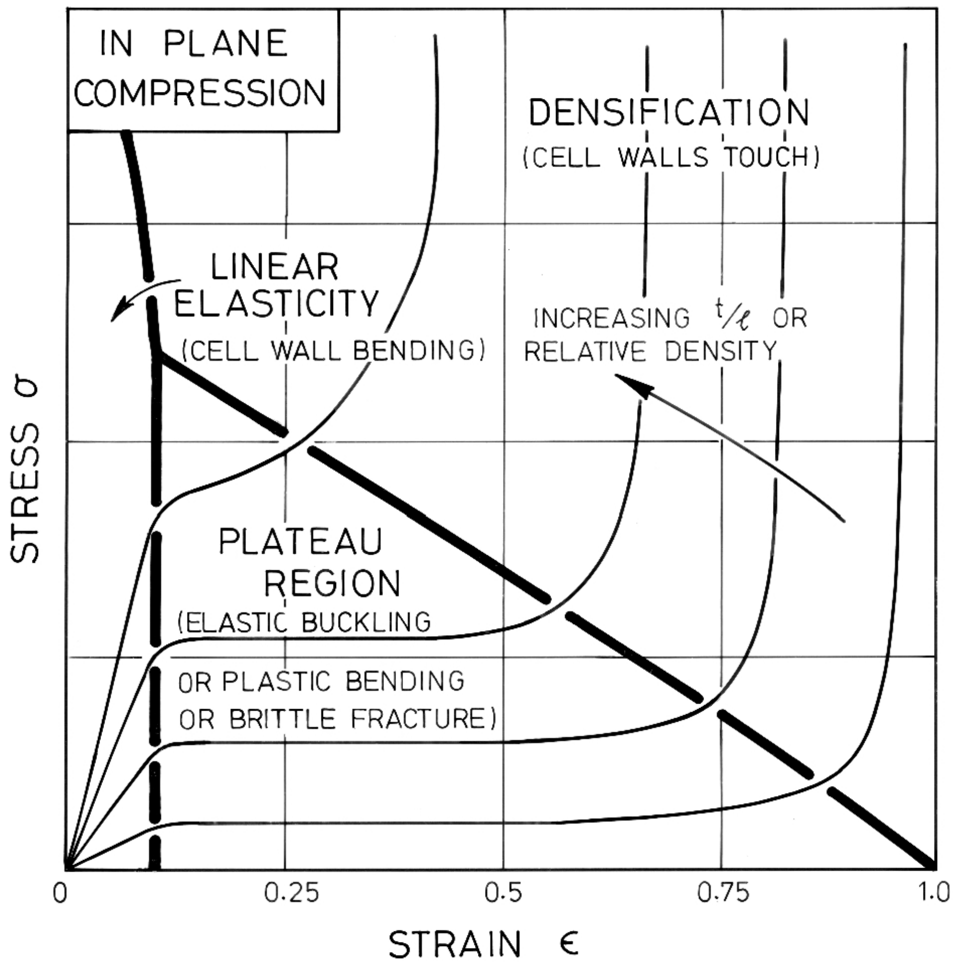
Buckling





Plastic collapse in an aluminum honeycomb

Stress-Strain Curve



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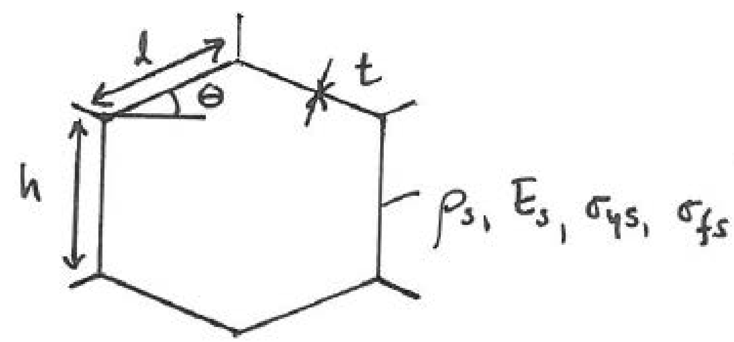
Tension

- Linear elastic $-$ bending
- Stress plateau
  - exists only if cell walls yield
  - no buckling in tension
  - brittle honeycombs fracture in tension

Variables affecting honeycomb properties

- Relative density $\quad \dfrac{\rho^*}{\rho_s} = \dfrac{\left(\frac{t}{l}\right)\left(\frac{h}{l}+2\right)}{2\,\cos\theta\,\left(\frac{h}{l}\sin\theta\right)} = \dfrac{2}{\sqrt{3}}\,\dfrac{t}{l} \quad$ regular hexagons
- Solid cell wall properties: ρ_s, E_s, σ_{ys}, σ_{fs}
- Cell geometry: h/l, θ



In-plane properties

Assumptions:

- t/l small ((ρ_c^*/ρ_s) small) — neglect axial and shear contribution to deformation
- Deformations small — neglect changes in geometry
- Cell wall — linear elastic, isotropic

Symmetry

- Honeycombs are orthotropic — rotate 180° about each of three mutually perpendicular axes and structure is the same

Linear elastic deformation

$$\begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\\ \epsilon_4\\ \epsilon_5\\ \epsilon_6\end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_2 & 0 & 0 & 0\\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0\\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & -1/E_3 & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G_{23} & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G_{13} & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G_{12}\end{bmatrix} \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix}$$

- Matrix notation:

$$\begin{array}{llll} \epsilon_1 = \epsilon_{11} & \epsilon_4 = \gamma_{23} & \sigma_1 = \sigma_{11} & \sigma_4 = \sigma_{23} \\ \epsilon_2 = \epsilon_{22} & \epsilon_5 = \gamma_{13} & \sigma_2 = \sigma_{22} & \sigma_5 = \sigma_{13} \\ \epsilon_3 = \epsilon_{33} & \epsilon_6 = \gamma_{12} & \sigma_3 = \sigma_{33} & \sigma_6 = \sigma_{12} \end{array}$$

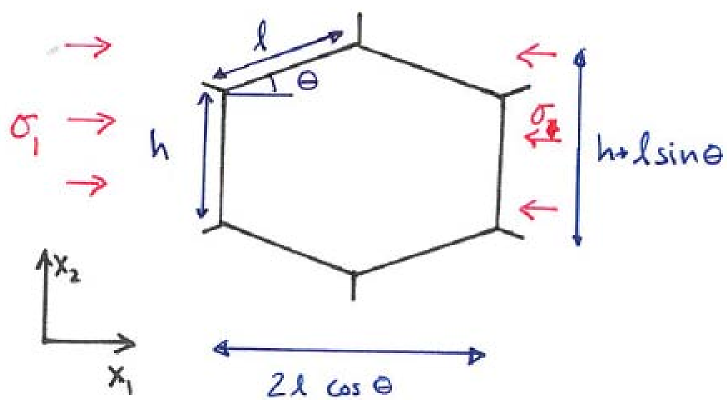
- In-plane $(x_1 - x_2)$: 4 independent elastic constants:

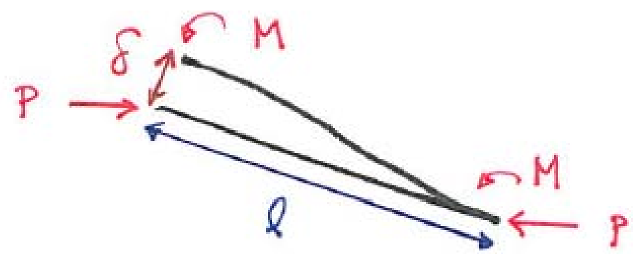
  $E_1 \quad E_2 \quad \nu_{12} \quad G_{12}$

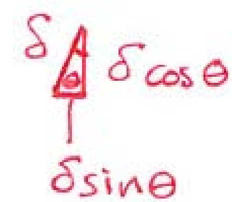
  and compliance matrix symmetric $\quad \dfrac{-\nu_{12}}{E_1} = \dfrac{-\nu_{21}}{E_2} \quad$ (reciprocal relation)

$$\left[\text{notation for Poisson's ratio:} \quad \nu_{ij} = \frac{-\epsilon_j}{\epsilon_i}\right]$$

Young's modulus in x_1 direction







Unit cell in x_1 direction: $2l\cos\theta$
Unit cell in x_2 direction: $h + 2l\sin\theta$

$$\sigma_1 = \frac{P}{(n + l\sin\theta)\,b}$$

$$\epsilon_1 = \frac{\delta\ \sin\theta}{l\ \cos\theta}$$

In-Plane Deformation: Linear Elasticity

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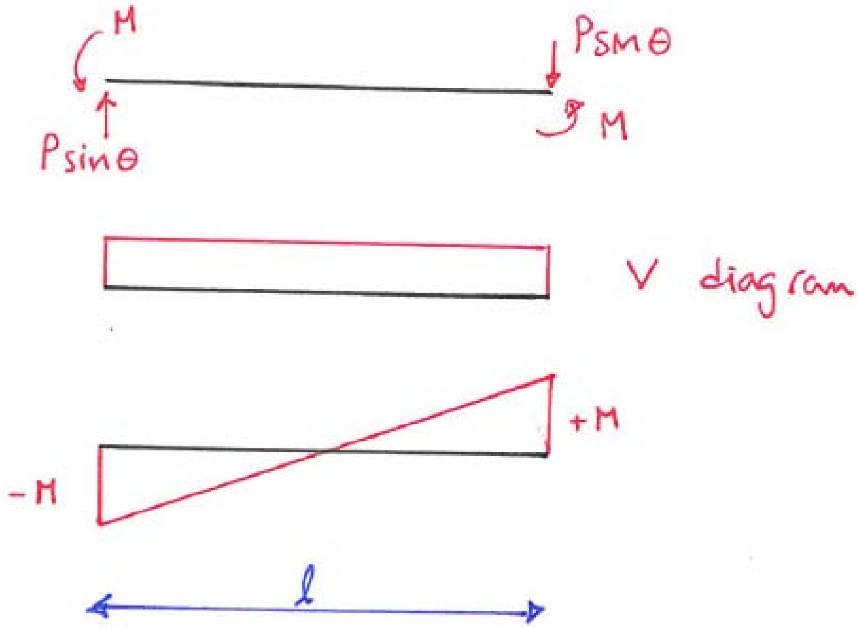


M diagram: 2 cantilevers of length l/2

$$\delta = 2 \cdot \frac{P \ \sin\theta (l/2)^3}{3 E_s \, I} = \frac{2 \, P l^3 \sin\theta}{24 \, E_s \, I}$$

$$\delta = \frac{P l^3 \sin\theta}{12 \, E_s \, I} \qquad I = \frac{b \, t^3}{12}$$

Combining:

$$E_1^* = \frac{\sigma_1}{\epsilon_1} = \frac{P}{(h + l \ \sin\theta) \, b} \, \frac{l \ \cos\theta}{\delta \sin\theta}$$

$$= \frac{P}{(h + l \ \sin\theta) \, b} \, \frac{l \ \cos\theta}{P \, l^3 \ \sin^2\theta} \, 12 \, E_s \, \frac{b \, t^3}{12}$$

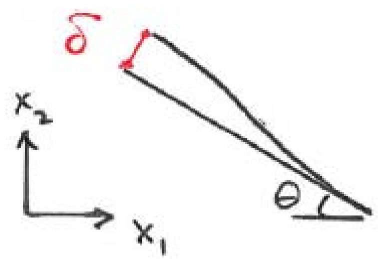
$$\boxed{E_1^* = E_s \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{(h/l + \sin\theta) \ \sin^2\theta}} = \frac{4}{\sqrt{3}} \left(\frac{t}{l}\right)^3 E_s$$

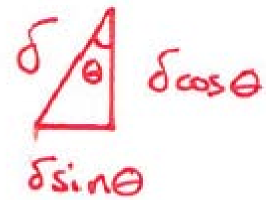
$\uparrow$ solid property $\quad \uparrow$ relative density $\quad \uparrow$ cell geometry

regular hexagons
h/l=1 $\theta = 30°$

Poisson's ratio for loading in x_1 direction

$$\nu_{12}^* = -\frac{\epsilon_2}{\epsilon_1}$$





$$\epsilon_1 = \frac{\delta \sin\theta}{l \ \cos\theta} \qquad \epsilon_2 = \frac{\delta\cos\theta}{h + l \ \sin\theta} \quad \text{(lengthens)}$$

$$\nu_{12}^* = \frac{\delta\cos\theta}{h + l \ \sin\theta}\left(\frac{l \ \cos\theta}{\delta\sin\theta}\right) = \frac{\cos^2\theta}{(h/l + \sin\theta) \ \sin\theta}$$

- ν_{12}^* depends <u>ONLY</u> on cell geometry (h/l, θ), not on E_s, t/l
- Regular hexagonal cells: $\nu_{12}^* = 1$
- ν can be negative for $\theta < 0$

  e.g. h/l=2 $\qquad \theta = -30^\circ \quad \nu_{12}^* = \frac{3/4}{(3/2)(-1/2)} = -1$

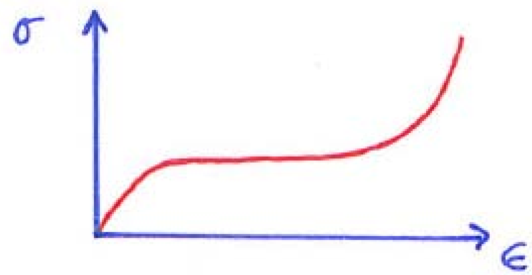
$\mathbf{E_2^*}$ $\quad \boldsymbol{\nu_{12}^*}$ $\quad \mathbf{G_{12}^*}$

- Can be found in similar way; results in book

Compressive strength (plateau stress)

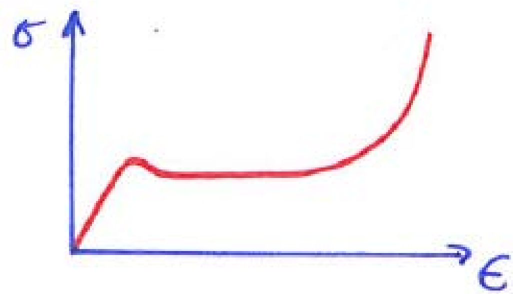
- Cell collapse by:

(1) elastic buckling



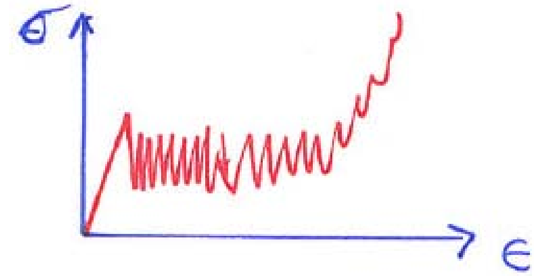
- buckling of vertical struts throughout honeycomb

(2) plastic yielding



- localization of yield
- as deformation progresses, propagation of failure band

(3) brittle crushing

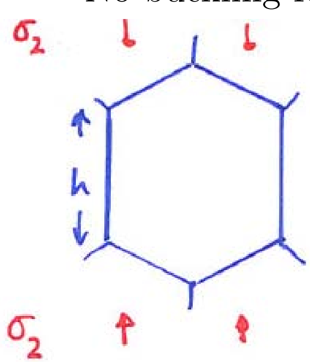


- peaks and valleys correspond to fracture of individual cell walls

Plateau stress: elastic buckling, σ^*_{el}

- Elastomeric honeycombs — cell collapse by elastic buckling of walls of length h when loaded in x_2 direction

- No buckling for σ_1; bending of inclined walls goes to densification



Euler buckling load

$$P_{cr} = \frac{n^2\,\pi^2\,E_s I}{h^2}$$

n=end constraint factor

pin-pin
n=1

fixed-fixed
n=2

Elastic Buckling

Figure removed due to copyright restrictions. See Figure 7: L. J. Gibson, M. F. Ashby, et al. "The Mechanics of Two-Dimensional Cellular Materials."

- Here, constraint n depends on stiffness of adjacent inclined members
- Can find elastic line analysis (see appendix if interested)
- Rotational stiffness at ends of column, h, matched to rotational stiffness of inclined members
- Find

n/l=1	1.5	2
n=0.686	0.760	0.860

and $$(\sigma^*_{el})_2 = \frac{P_{cr}}{2l\cos\theta\, b} = \frac{n^2\,\pi^2\,E_s}{h^2\,2l\,\cos\theta\, b}\,\frac{bt^3}{12}$$

$$\boxed{(\sigma^*_{el})_2 = \frac{n^2\pi^2}{24}\,E_s\,\frac{(t/l)^3}{(h/l)^2\,\cos\theta}}$$

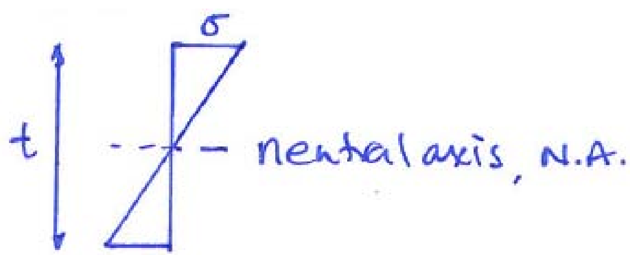
regular hexagons: $(\sigma^*_{el})_2 = 0.22\,E_s(t/l)^3$

and since $E^*_2 = 4/\sqrt{3}\,E_s(t/l)^3 = 2.31\,E_s(t/l)^3$

strain at buckling $(\epsilon^*_{el})_2 = 0.10$, for regular hexagons, independent of E_s, t/l

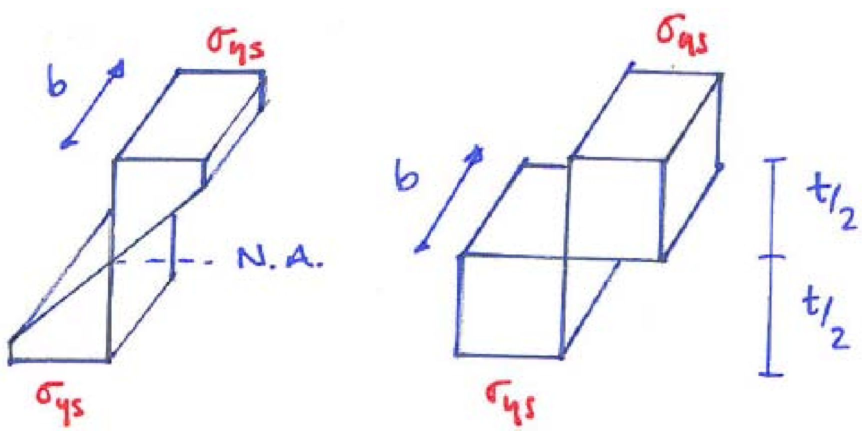
Plateau stress: plastic yielding, σ^*_{pl}

- Failure by yielding in cell walls
- Yield strength of cell walls $= \sigma_{ys}$
- Plastic hinge forms when cross-section fully yields
- Beam theory — linear elastic $\sigma = \frac{My}{I}$



- Once stress outer fiber=σ_{ys}, yielding begins and then progresses through the section, as the load increases

as P ↑



- When section fully yielded (right figure), form plastic "hinge"
- Section rotates like a pin

Plastic Collapse

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- Moment at formation of plastic hinge (plastic moment, M_p):

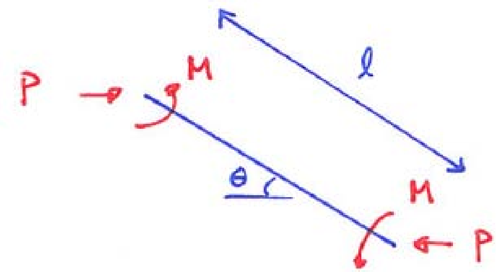
$$M_p = \left(\sigma_{ys} \frac{b\,t}{2}\right)\left(\frac{t}{2}\right) = \frac{\sigma_{ys}\, b\, t^2}{4}$$

- Applied moment, from applied stress

$$2M_{app} - PL\sin\theta = 0$$

$$M_{app} = \frac{P\,l\sin\theta}{2}$$

$$\sigma_1 = \frac{P}{(h + l\sin\theta)\,b}$$



$$M_{app} = \sigma_1\,(h + l\sin\theta)\,b\,\frac{l\sin\theta}{2}$$

- Plastic collapse of honeycomb at $(\sigma^*_{pl})_1$, when $M_{app} = M_p$

$$(\sigma^*_{pl})_1\,(h + l\sin\theta)\,\cancel{b}\,\frac{l\sin\theta}{\cancel{2}} = \sigma_{ys}\,\frac{\cancel{b}\,t^2}{\cancel{4}\,2}$$

$$\boxed{(\sigma^*_{pl})_1 = \sigma_{ys}\left(\frac{t}{l}\right)^2 \frac{1}{2(h/l + \sin\theta)\sin\theta}}$$

regular hexagons: $(\sigma^*_{pl})_1 = \dfrac{2}{3}\,\sigma_{ys}\left(\dfrac{t}{l}\right)^2$

similarly, $(\sigma^*_{pl})_2 = \sigma_{ys}\left(\dfrac{t}{l}\right)^2 \dfrac{1}{2\cos^2\theta}$

- For thin-walled honeycombs, elastic buckling can precede plastic collapse (for σ_2)
- Elastic buckling stress = plastic collapse stress $(\sigma^*_{el})_2 = (\sigma^*_{pl})_2$

$$\frac{n^2\pi^2}{24}\, E_s \frac{(t/l)^3}{(h/l)^2\ \cos\theta} = \frac{\sigma_{ys}(t/l)^2}{2\ \cos^2\theta}$$

$$(t/l)_{\text{critical}} = \frac{12\,(h/l)^2}{n^2\,\pi^2\cos\theta}\left(\frac{\sigma_{ys}}{E_s}\right)$$

$$\text{regular hexagons:} (t/l)_{\text{critical}} = 3\,\frac{\sigma_{ys}}{E_s}$$

- E.g. metals $\sigma_{ys}/E_s \sim .002$ $(t/l)_{\text{critical}} \sim 0.6\%$
  - most metal honeycomb denser than this
    polymer $\sigma_{ys}/E_s \sim 3-5\%$ $(t/l)_{\text{critical}} \sim$ 10-15%
  - low density polymers with yield point may buckle before yield

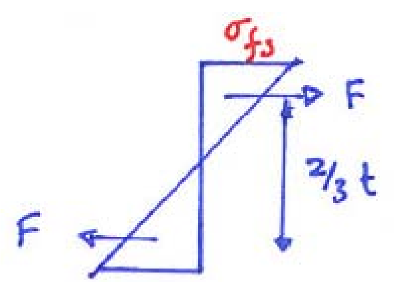
Plastic stress: brittle crushing, $(\sigma_{cr}^{*})_1$

- Ceramic honeycombs — fail in brittle manner
- Cell wall bending — stress reaches modulus of rapture — wall fracture loading in x_1 direction:

$P = \sigma_1 \,(h + l \sin\theta)\, b \qquad \sigma_{fs}$ = modulus of rupture of cell wall

$$M_{\text{max. applied}} = \frac{P\, l\, \sin\theta}{2} = \frac{\sigma_1\,(h + l\sin\theta)\, b\, l \sin\theta}{2}$$

Moment at fracture, M_f

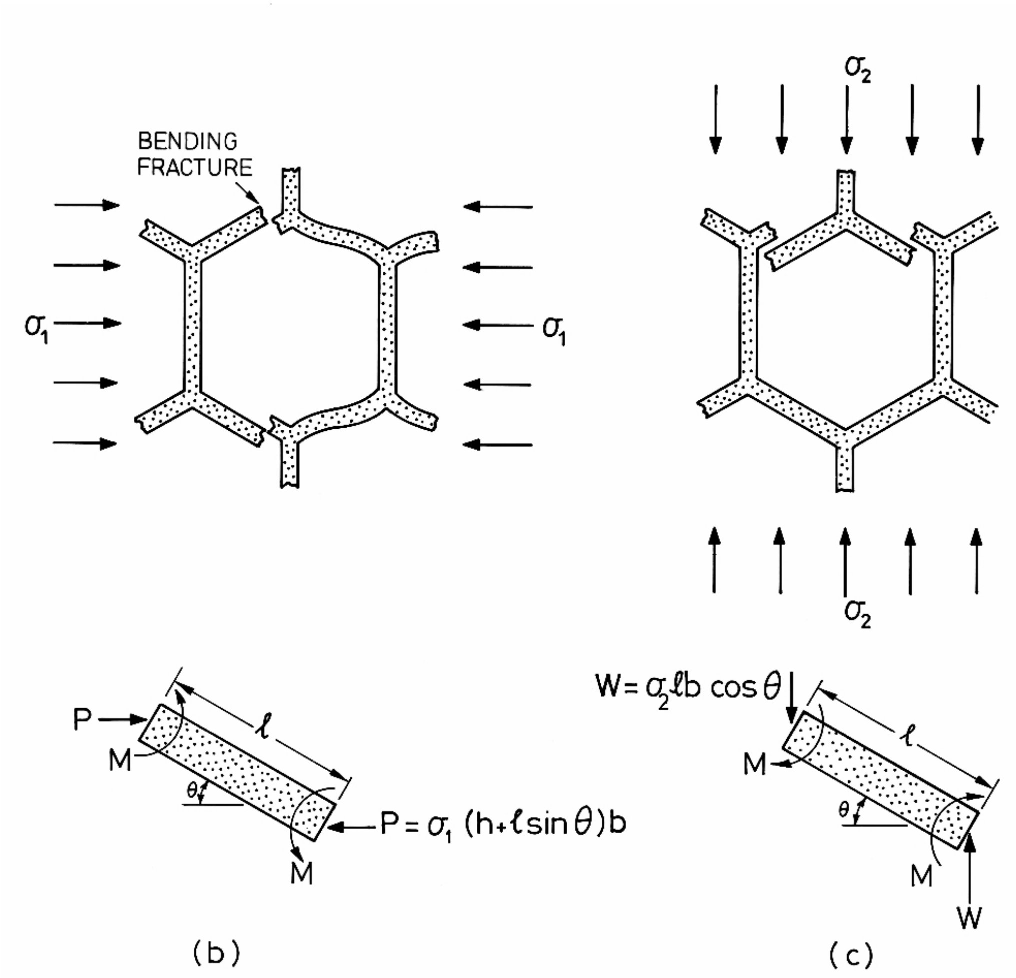


$$M_f = \left(\frac{1}{2}\,\sigma_{fs}\, b\, \frac{t}{2}\right)\left(\frac{2}{3}\, t\right) = \frac{\sigma_{fs}\, b\, t^2}{6}$$

$$\boxed{(\sigma_{cr}^{*})_1 = \sigma_{fs}\left(\frac{t}{l}\right)^2 \frac{1}{3\,(h/l + \sin\theta)\sin\theta}}$$

regular hexagons: $(\sigma_{cr}^{*})_1 = \dfrac{4}{9}\,\sigma_{fs}\left(\dfrac{t}{l}\right)^2$

Brittle Crushing



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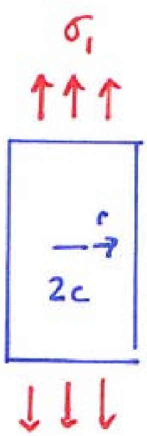
Tension

- No elastic buckling
- Plastic plateau stress approx. same in tension and compression (small geometric difference due to deformation)
- Brittle honeycombs: fast fracture

Fracture toughness

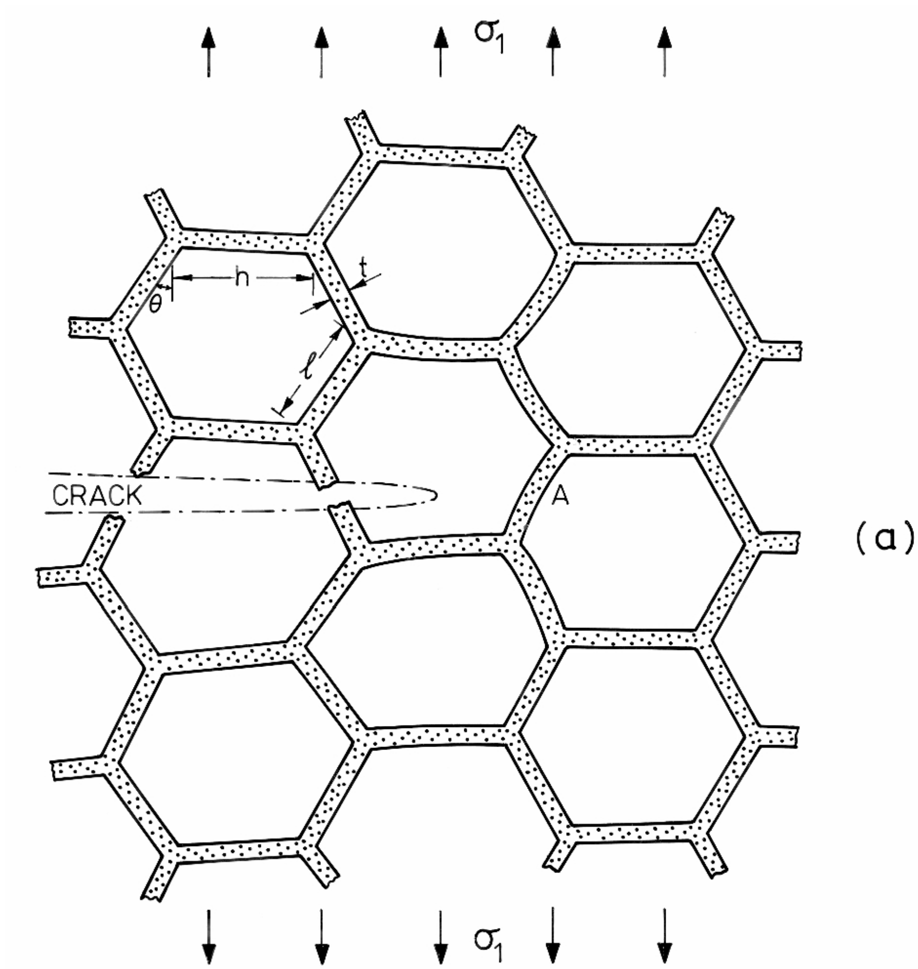
Assume:
- crack length large relative to cell size (continuum assumption)
- axial forces can be neglected
- cell wall material has constant modulus of rapture, σ_{fs}

Continuum: crack of length 2c in a linear elastic solid material normal to a remote tension stress σ_1 creates a local stress field at the crack tip



$$\sigma_{\text{local}} = \sigma_l = \frac{\sigma_1 \sqrt{\pi\, c}}{\sqrt{2\pi\, r}}$$

Fracture Toughness



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Honeycomb: cell walls bent — fail when applied moment = fracture moment

$M_{app} \propto P\, l$ on wall A

$$M_{app} \propto P\, l \propto \sigma_l\, l^2\, b \propto \frac{\sigma_1 \sqrt{c}\, l^2\, b}{\sqrt{l}} \propto \sigma_{fs}\, b\, t^2$$

$$(\sigma_f^*)_1 \propto \sigma_{fs} \left(\frac{t}{l}\right)^2 \sqrt{\frac{l}{c}}$$

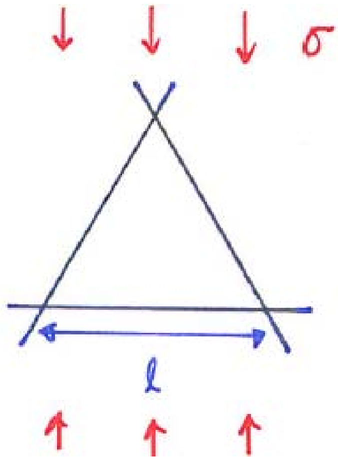
$$\boxed{K_{IC}^* = \sigma_f^* \sqrt{\pi c} = c\, \sigma_{fs} \left(\frac{t}{l}\right)^2 \sqrt{l}}$$ depends on cell size, l!

c=constant

Summary: hexagonal honeycombs, in-plane properties

- Linear elastic moduli: E_1^* E_2^* ν_{12}^* G_{12}^*
- Plateau stresses (compression)
  - $(\sigma_{el}^*)_2$ elastic buckling
  - σ_{pl}^* plastic yield
  - σ_{cr}^* brittle crushing
- Fracture toughness (tension) K_{IC}^* brittle fracture

Honeycombs: In-plane behavior — triangular cells



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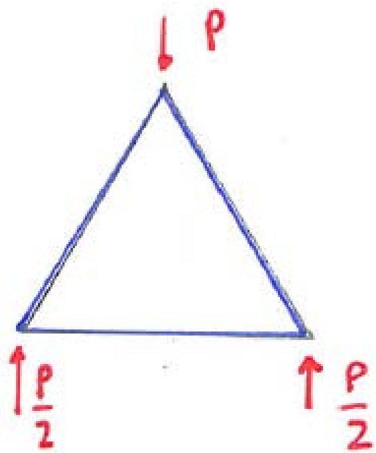
- Triangulated structures - trusses
- Can analyze as pin-jointed (no moment at joints)
- Forces in members all axial (no bending)
- If joints fixed and include bending, difference $\sim 2\%$
- Force in each member proportional to P



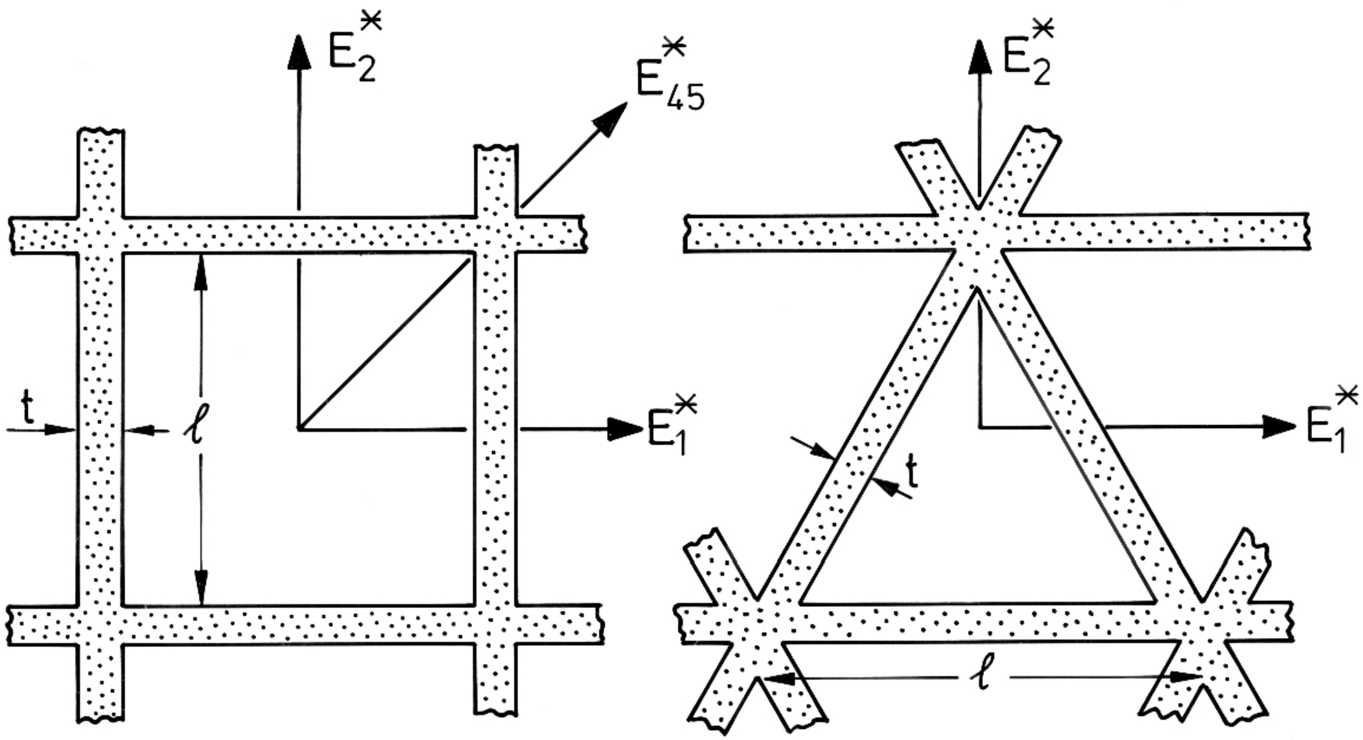
$$\sigma \propto \frac{P}{l\,b} \qquad \epsilon \propto \frac{\delta}{l} \qquad \delta \propto \frac{P\,l}{A\,E_s} \text{axial shortening: Hooke's law}$$

$$E^* \propto \frac{\sigma}{\epsilon} \propto \frac{P}{l\,b}\,\frac{l}{\delta} \propto \frac{P}{b}\,\frac{b\,t\,E_s}{P\,l} \propto E_s\left(\frac{t}{l}\right)$$

$$E^* = c\,E_s\,(t/l)$$

exact calculation: $E^* = 1.15\,E_s\,(t/l)$ for equilateral triangles

Square and Triangular Honeycombs



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