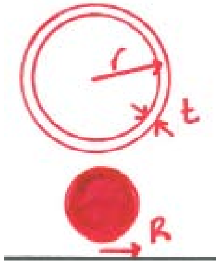
Lecture 8, Foam Design Notes, 3.054

Foams: Microstructural Design

- Foam—behavior dominated by cell wall bending
- Foam properties can be increased by increasing EI of cell walls

Hollow walls



Thin walled tube $I_t = \pi\, r^3 t$

Solid circular section $I_s = \frac{\pi\, R^4}{4}$

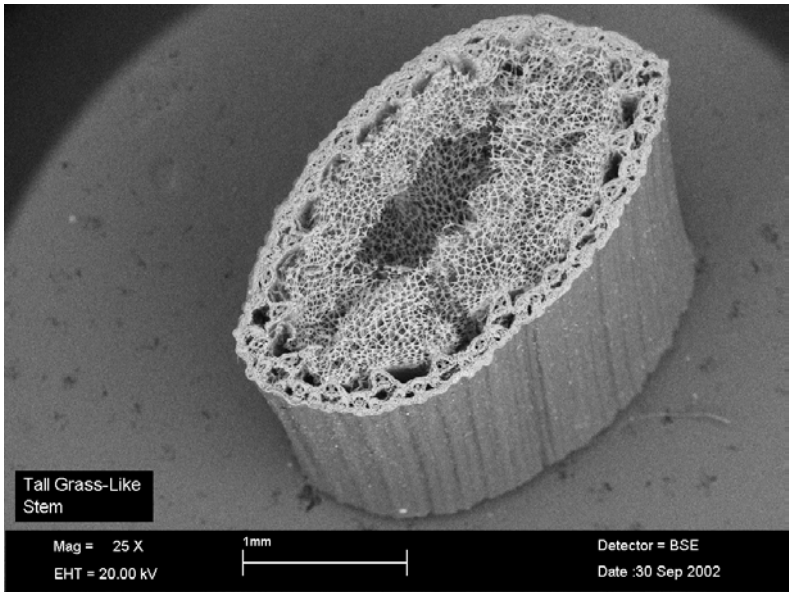
Masses equal: $\pi R^2 = 2\pi r t$

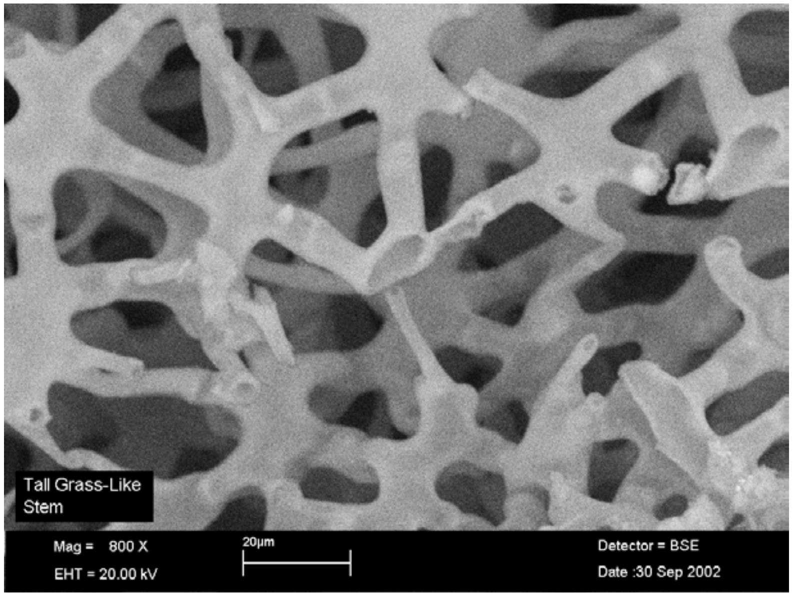
$$R = \sqrt{2rt}$$

$$\frac{I_t}{I_s} = \frac{4\pi r^3 t}{\pi R^4} = \frac{4 r^3 t}{4 r^2 t^2} = \frac{r}{t}$$

$$\therefore \quad \frac{E^*_{\text{tube wall}}}{E^*_{solidwall}} \propto \frac{r}{t}$$

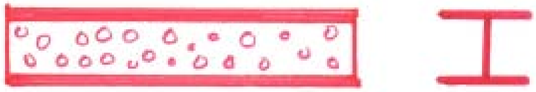
- Can do similar analysis for other properties


Tall Grass-Like Stem
Mag = 25 X
EHT = 20.00 kV
1mm
Detector = BSE
Date :30 Sep 2002


Tall Grass-Like Stem
Mag = 800 X
EHT = 20.00 kV
20µm
Detector = BSE
Date :30 Sep 2002

Sandwich cell walls

- Sandwich beam — two stiff faces separated by a lightweight core
  — core typically a honeycomb or foam (or balsa)



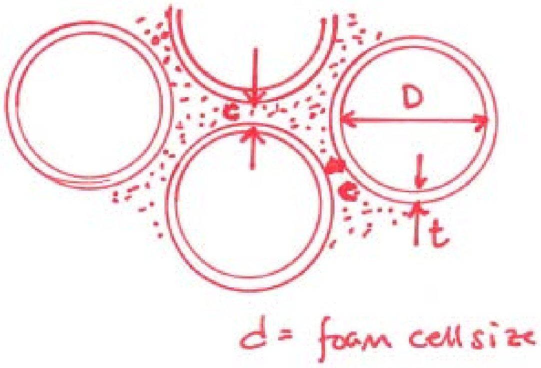
I-beam effect - increase in moment of inertia, with little increase in weight

Faces — like flanges of I-beam — resist bending

Core — like web of I-beam — resist shear

- Microsandwich foam

Microsandwich foam



- Thin walled hollow spheres distributed in a foam
- Have to get geometry right
- Require:

Spheres: $t \ll D$

$E_{\text{sphere}} \gg E_{\text{foam}}$

$V_{\text{f sphere}} \approx 50-60\%$

Foam: $d \ll c$

Foams: Microstructural Design

- Another alternative is to use microstructure that induces axial, rather than bending, deformations
- 3D lattice materials: — triangulated trusses in 3D
  — Forces in members all axial; bending negligible
- Various processing methods and geometries possible - all triangulated
- Can analyze truss as having pin joints - axial forces in members
- Open-cell structures

$$E^* = \frac{\sigma}{\epsilon} \propto \frac{F}{l^2}\,\frac{l}{\delta} \qquad \delta \propto \frac{Fl}{AE_s} \propto \frac{Fl}{t^2\,E_s}$$

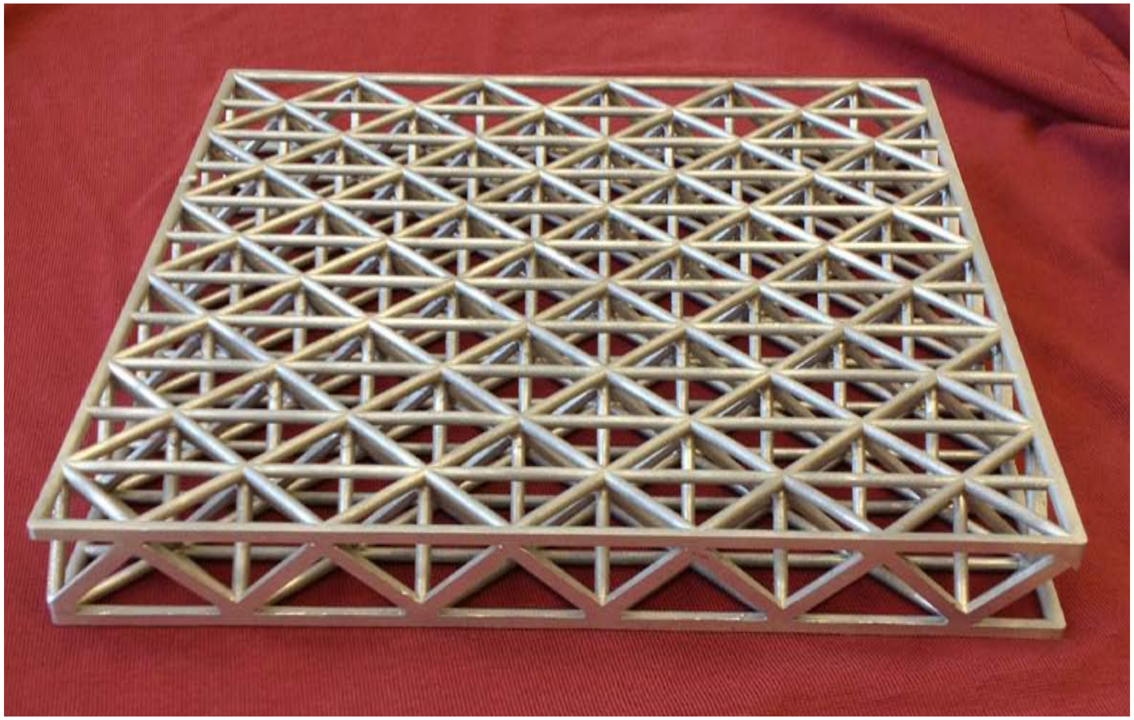
$$E^* \propto \frac{F}{l}\,\frac{E_s t^2}{Fl} \propto E_s\left(\frac{t}{l}\right)^2 \propto E_s\left(\frac{\rho^*}{\rho_s}\right)$$

$$\boxed{E^* = C\,E_s(\rho^*/\rho_s)}$$ C depends on cell geometry and loading direction

- Strength: if struts fail by *uniaxial yield*:

$$\sigma^*_{\mathrm{pl}} = C\,\sigma_{ys}(\rho^*/\rho_s)$$

But some struts in compression - may *buckle* (generally do buckle)



Compressive strut buckling

- Elastic buckling $\sigma^*_{\text{el}} = CE_s\,(\rho^*/\rho_s)^2$ (like open-cell foam)
- If interaction between elastic buckling and yield — use a reduced modulus (tangent modulus)
- Also: Imperfections such as non-straight struts or misaligned struts reduce buckling resistance
  "knock-down" factor can be significant $\sim 50\%$

Material Selection

- How to select the best material for same mechanical requirement?
- Section on wood: derived performance index for minimizing mass of a beam of a given stiffness: $\boxed{E^{1/2}/\rho}$
- Here, discuss material selection more broadly
- Another example: What material minimizes the mass of a beam of a given failure load, P_f? Given P_f, span l, square cross-section t^2

$$\sigma_{max} = \frac{M\,y}{I}$$

$$\sigma_{max} \propto \frac{P_f\,l\,t}{t^4} \propto \sigma_f$$

$$t \propto \left(\frac{P_f l}{\sigma_f}\right)^{1/3}$$

M = maximum moment in beam $\propto Pl$

y = maximum distance from neutral axis $\propto t$

I = moment of inertia $\propto t^4$

σ_f = failure stress of beam material

Mass, $m = \rho t^2 l$

$$m \propto \rho \left(\frac{P_f l}{\sigma_f}\right)^{2/3} l$$

Performing index: $\boxed{\sigma_f^{2/3}/\rho}$
to be maximized

Material Selection

Ashby book-Tables

- Can obtain performance indices for various loading configurations and mechanical requirements
- If plot data for material properties on log-log scales, performance indices appear as straight lines
- Shifting lines up and down identifies best material for that performance index
- Example: modulus-density chart

CMNM Fig.7.1

E/ρ: axially loaded tie of given stiffness

$E^{1/2}/\rho$: beam of given stiffness

$E^{1/3}/\rho$: plate of given stiffness

Property charts for foams:

E^* **vs** ρ^***:** range of E^* factor of 10^6, from 0.01 MPa to 10 GPa

σ^* **vs** ρ^***:** range of σ^* from 10^{-3} to 30 MPa
→ scope for matching foam properties to design requirements

CS Fig.13.1 Fig.13.2

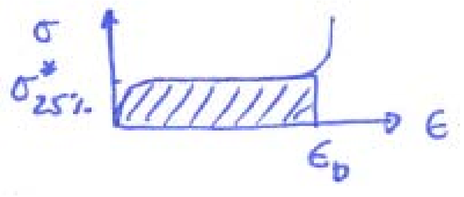
E^*/ρ **vs** σ^*/ρ**:** end grain balsa, metal foam high values
useful for sandwich panels - selection of core material

Property charts for foams

ϵ_D vs $\sigma^*_{25\%}$: contours show energy absorption/volume, U

$$U \sim \sigma^*_{25\%}\, \epsilon_D$$

$$\epsilon_D = U/\sigma^*_{25\%}$$

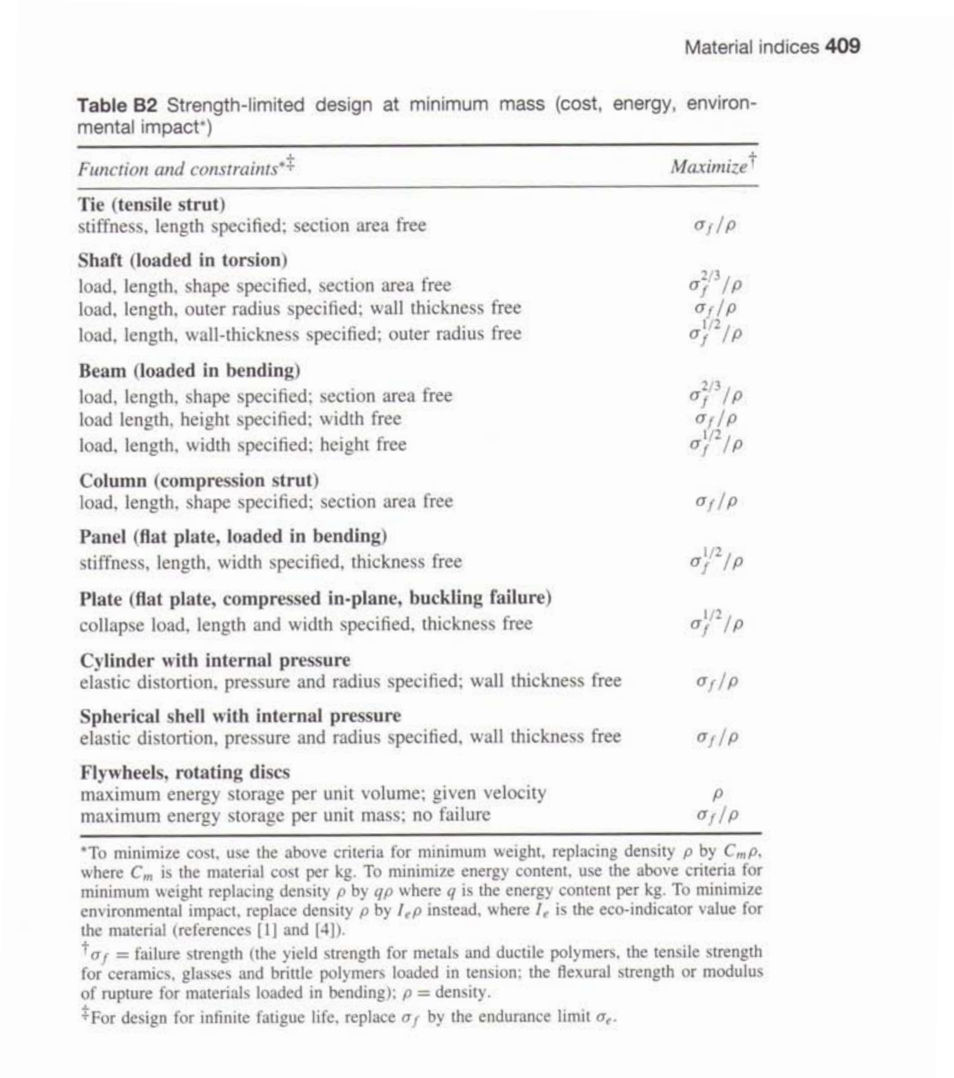


Contours have slope of -1 on log-log scales
Balsa, metal foams - high values of U

- Can also produce selection charts for other properties — e.g. thermal
- λ vs $\sigma^*_{5\%}$ — thermal conductivity, λ
  — thermal insulation applications usually have constraint on strength, too
- λ vs $T_{\max}$ — may have constraint on maximum service temperature, too
- Density plot — closed cell foams — buoyancy
- Cell size — open cell foams - filtration and catalysts
  — surface are/volume increases as cell size decreases
  e.g. ceramic foams used in filtration of liquid metals

Table removed due to copyright restrictions. See Table B1: Ashby, M. F. *Materials Selection in Mechanical Design*. 2nd ed. Butterworth Heinemann, 1999.

Ashby MF (1999) Materials Selection in Mechanical Design.
Second Edition Butterworth Heinemann



Material indices **409**

Table B2 Strength-limited design at minimum mass (cost, energy, environmental impact*)

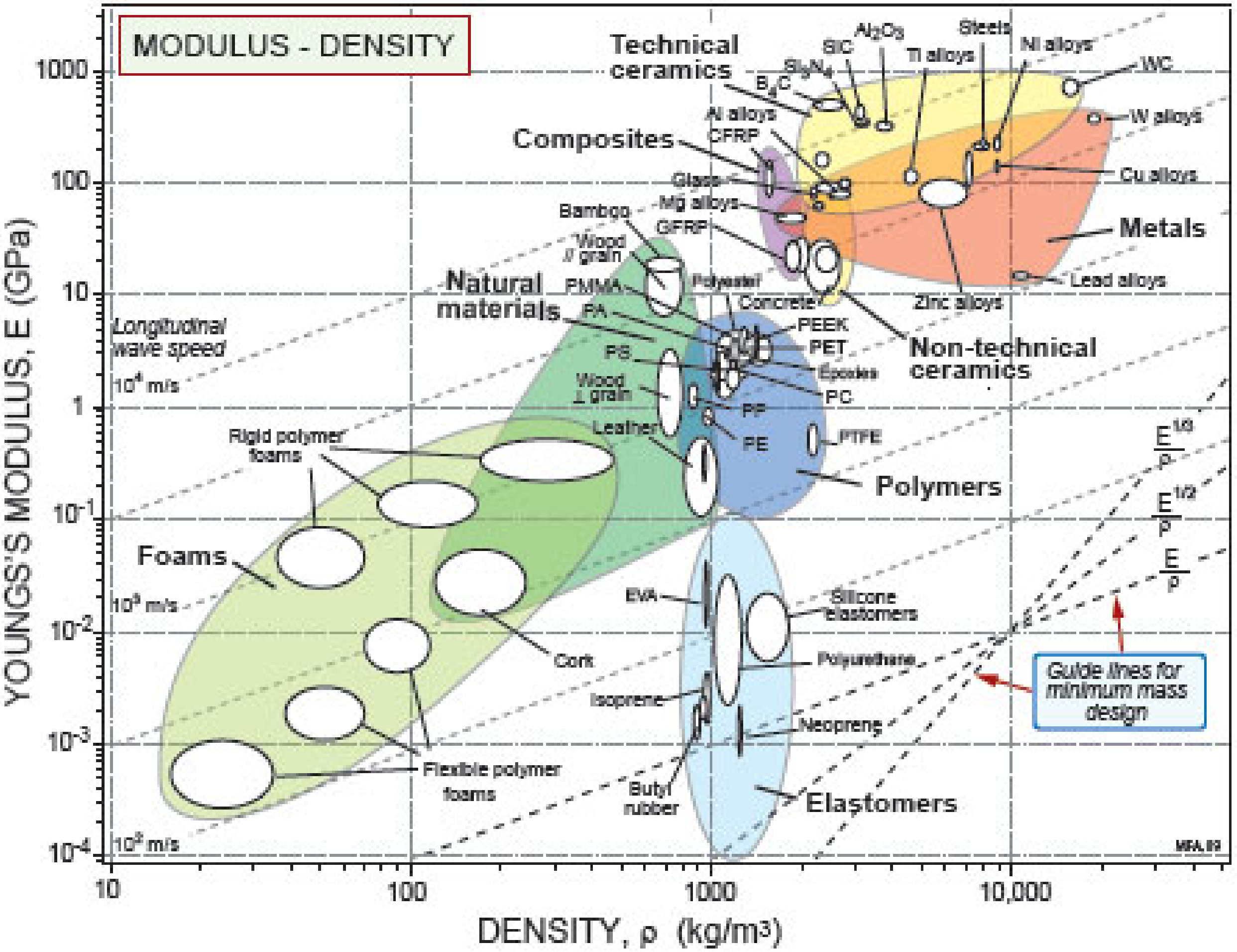
*Function and constraints**‡	*Maximize*†
Tie (tensile strut)	
stiffness, length specified; section area free	σ_f/ρ
Shaft (loaded in torsion)	
load, length, shape specified, section area free	$\sigma_f^{2/3}/\rho$
load, length, outer radius specified; wall thickness free	σ_f/ρ
load, length, wall-thickness specified; outer radius free	$\sigma_f^{1/2}/\rho$
Beam (loaded in bending)	
load, length, shape specified; section area free	$\sigma_f^{2/3}/\rho$
load length, height specified; width free	σ_f/ρ
load, length, width specified; height free	$\sigma_f^{1/2}/\rho$
Column (compression strut)	
load, length, shape specified; section area free	σ_f/ρ
Panel (flat plate, loaded in bending)	
stiffness, length, width specified, thickness free	$\sigma_f^{1/2}/\rho$
Plate (flat plate, compressed in-plane, buckling failure)	
collapse load, length and width specified, thickness free	$\sigma_f^{1/2}/\rho$
Cylinder with internal pressure	
elastic distortion, pressure and radius specified; wall thickness free	σ_f/ρ
Spherical shell with internal pressure	
elastic distortion, pressure and radius specified, wall thickness free	σ_f/ρ
Flywheels, rotating discs	
maximum energy storage per unit volume; given velocity	ρ
maximum energy storage per unit mass; no failure	σ_f/ρ

*To minimize cost, use the above criteria for minimum weight, replacing density ρ by $C_m\rho$, where C_m is the material cost per kg. To minimize energy content, use the above criteria for minimum weight replacing density ρ by $q\rho$ where q is the energy content per kg. To minimize environmental impact, replace density ρ by $I_e\rho$ instead, where I_e is the eco-indicator value for the material (references [1] and [4]).

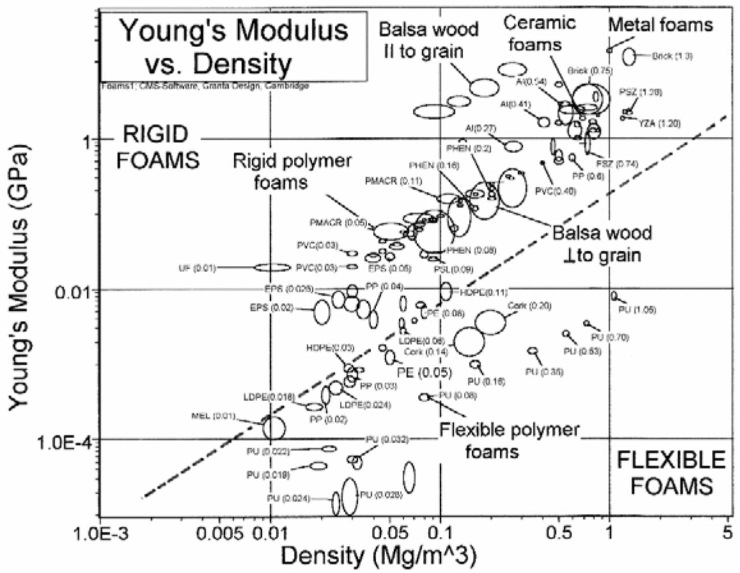
†σ_f = failure strength (the yield strength for metals and ductile polymers, the tensile strength for ceramics, glasses and brittle polymers loaded in tension; the flexural strength or modulus of rupture for materials loaded in bending); ρ = density.

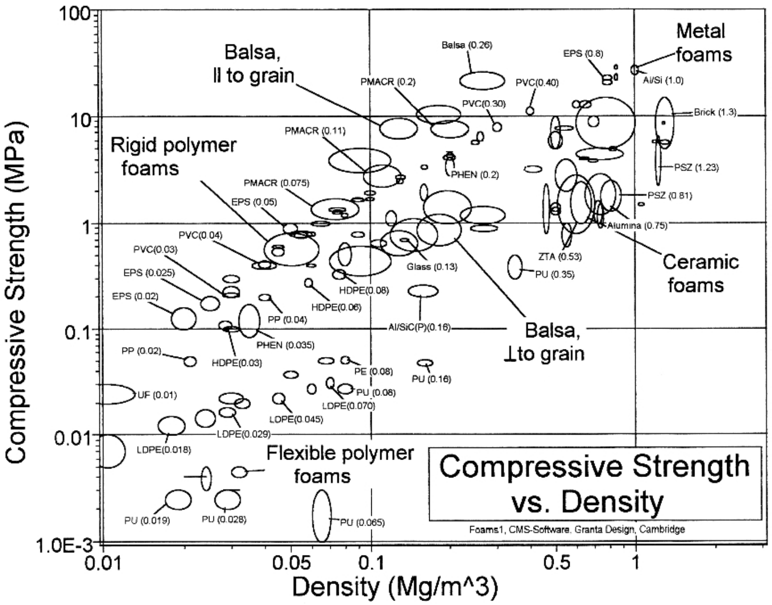
‡For design for infinite fatigue life, replace σ_f by the endurance limit σ_e.

Ashby MF (1999) Materials Selection in Mechanical Design. Second Edition Butterworth Heinemann

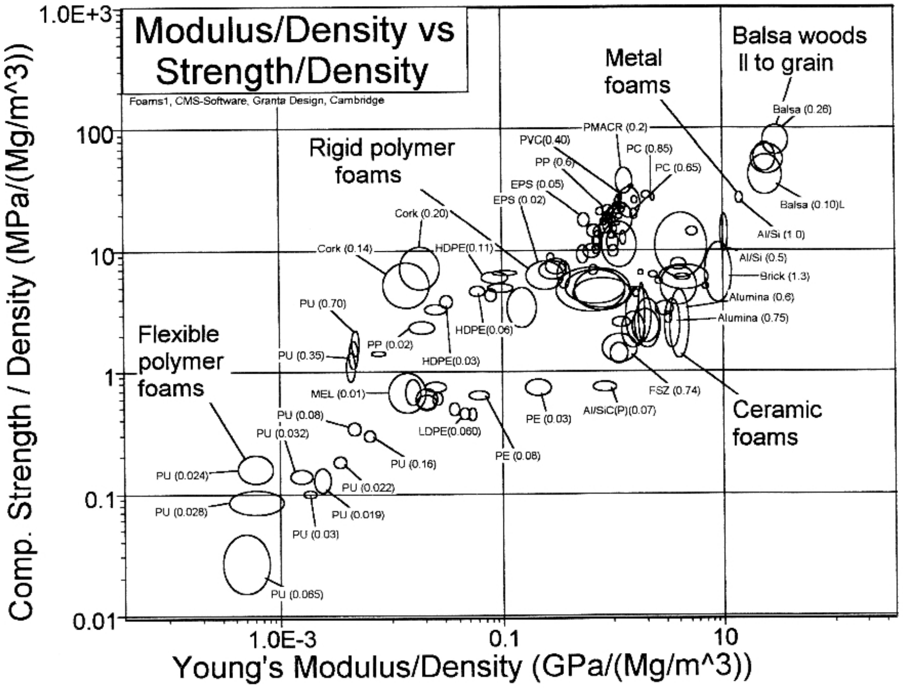


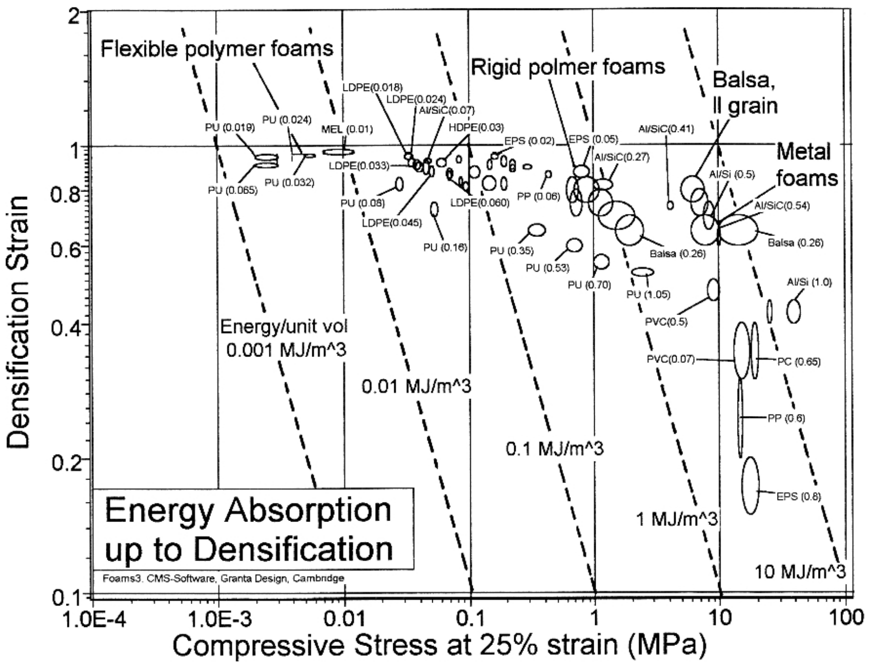
Ashby plot of Young's Modulus - Density © Granta Design. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/help/faq-fair-use/.





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.





End grain balsa; metal foams
Useful for sandwich panels

Contours show energy absorption per unit volume

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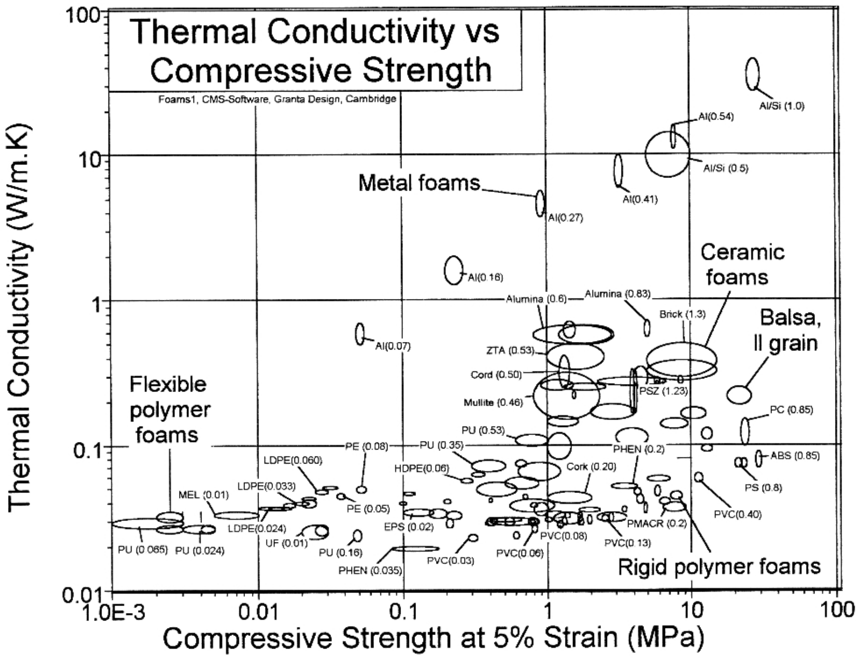

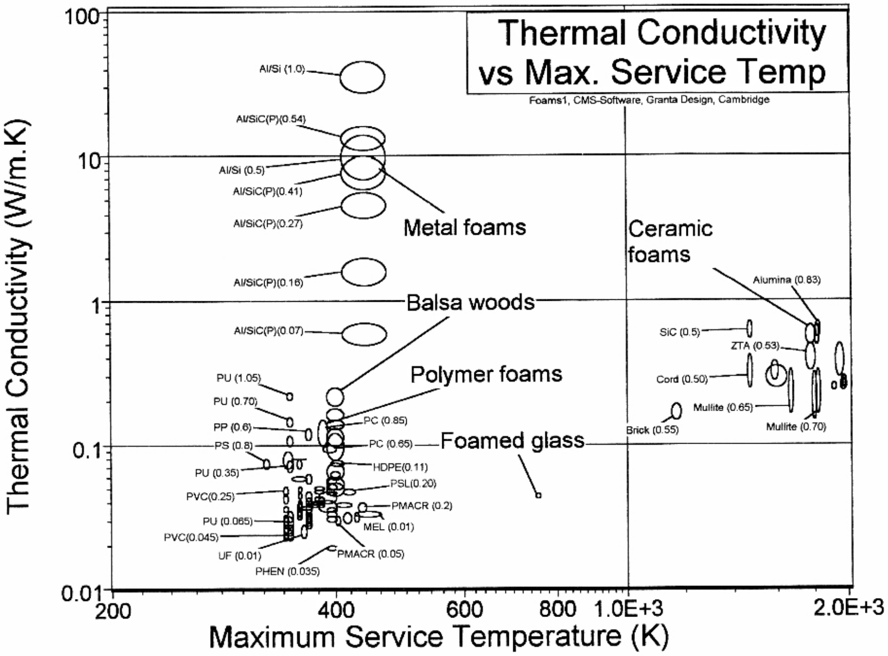

Thermal insulation applications;
Usually a constraint on strength, too

May also have a constraint on
maximum service temperature

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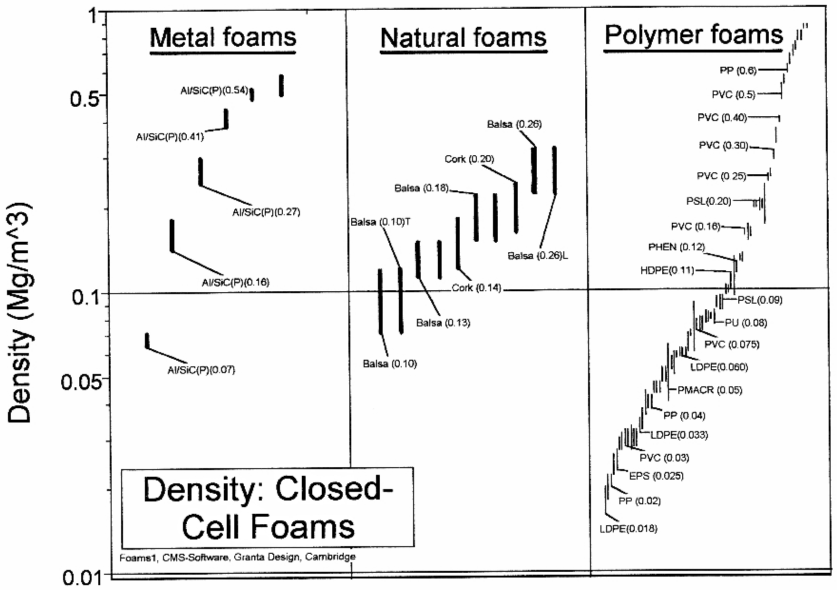

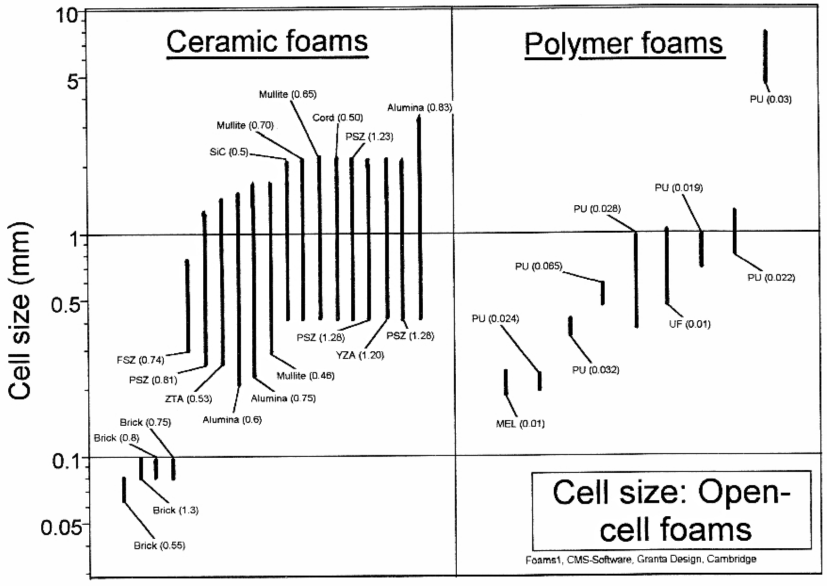

Buoyancy

Filtration and catalysis

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Spring 2014