Sandwich structures in nature

- previously, saw sandwich structures efficient in resisting bending, buckling
- sandwich panels also appear in nature:
  - leaves of monocotyledon plants (grasses, corn, iris)
  - skulls (esp. birds)
  - shells of some arthropods (eg. horseshoe crab)
  - cuttlefish bone (mollusk)

Leaves

- leaves must provide for structural support as well as large surface area for photosynthesis
- iris, cattail, ryegrass, giant feather grass - leaves all sandwich str.

Iris leaves

- nearly fully dense ribs (sclerenchyma) running along length of outer surfaces
- ribs separated by a core of foam-like parenchyma cells

Leaves

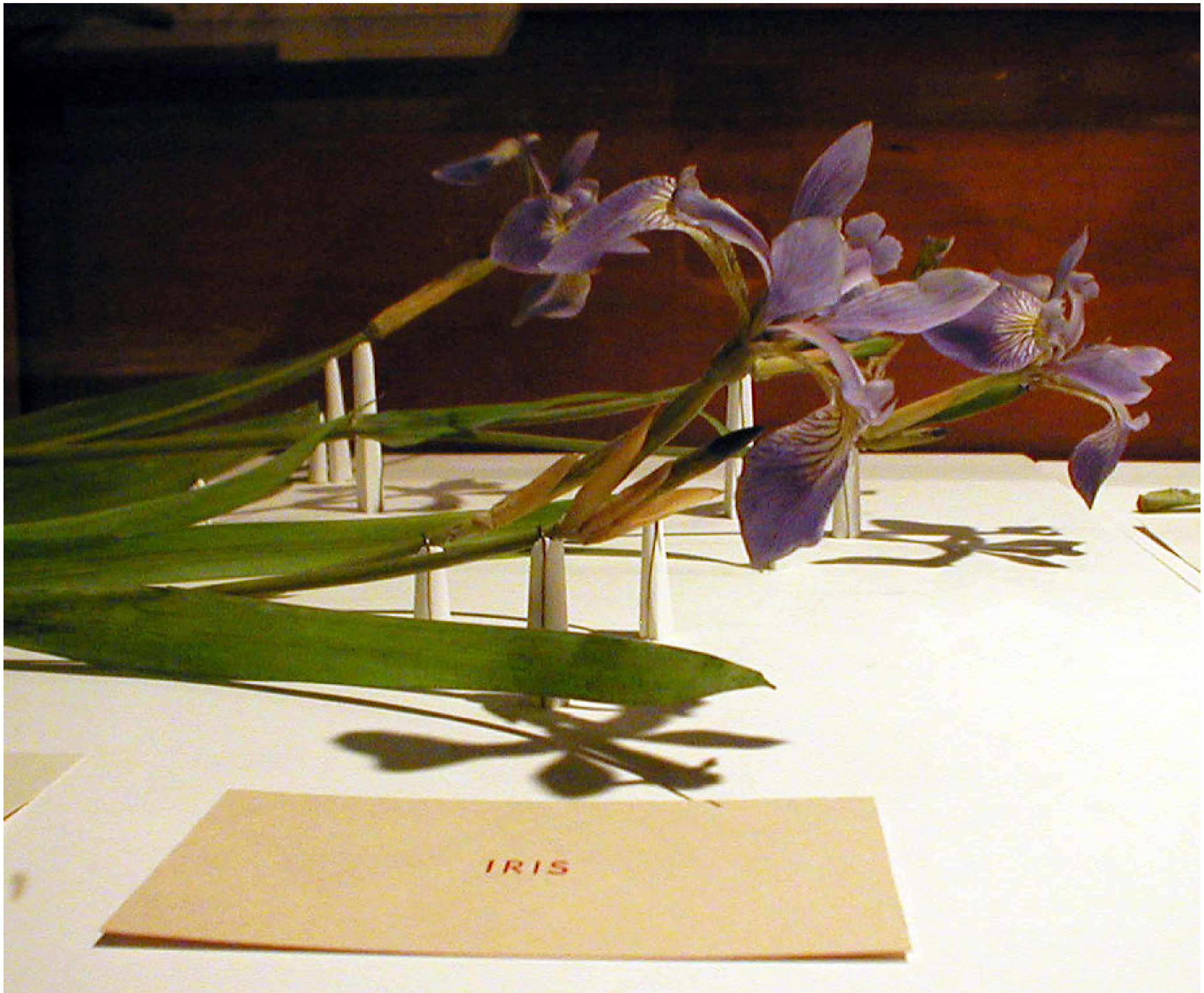
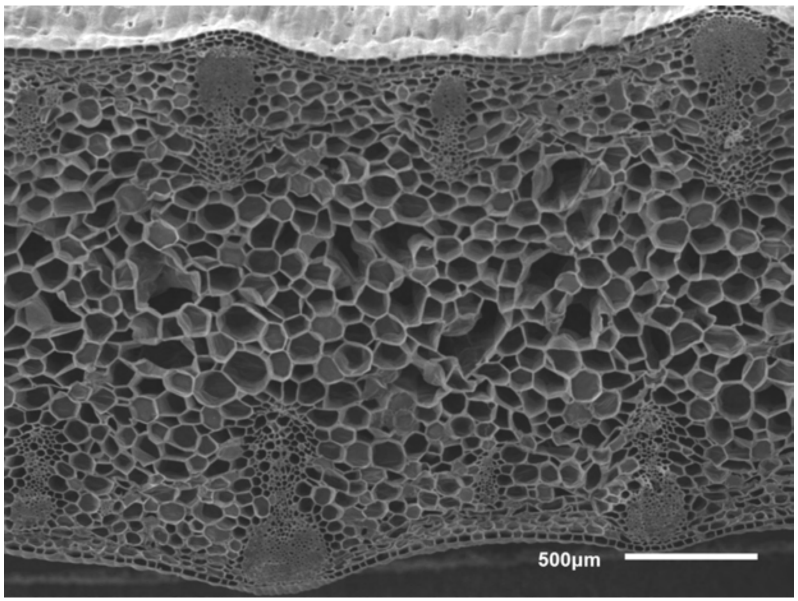
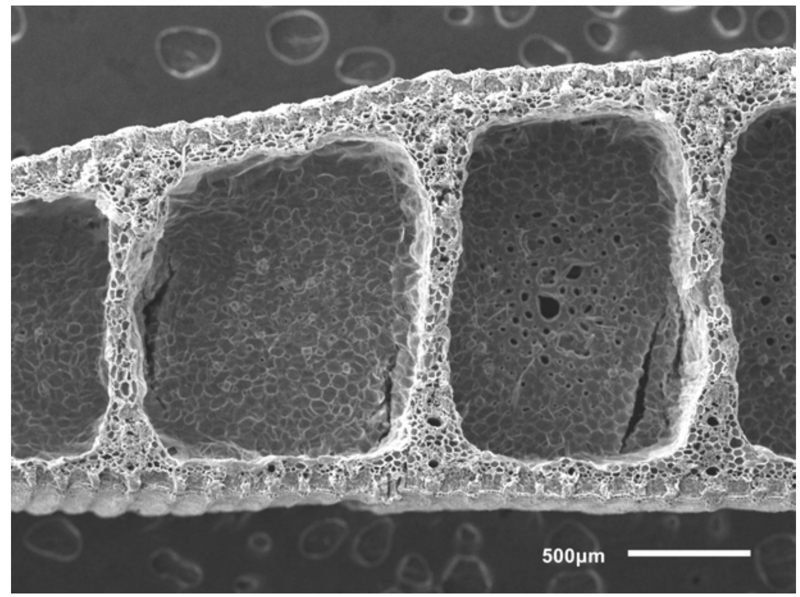


Photo of Blaschka glass flowers (iris) at the Harvard Museum of Natural History. Courtesy of Andrew Kuchling on Flickr.

Leaves



Iris



Cattail
(Bulrush)

Leaves

Iris leaf

Figures removed due to copyright restrictions. See Figures 3 and 4: Gibson, L. J., M. F. Ashby, et al. "Structure and Mechanics of the Iris Leaf." *Journal of Material Science* 23 (1988): 3041-48.

- outer face - ribs connected by single layer of roughly square cells
  - jointly act as fibre reinforced composite
- measurements of leaf microstructure summarized in Table
- can analyze leaf as a sandwich structure
- compare analysis with bending tests on fresh iris leaves
  - cantilevers with weights hung from free end ($B_1 = 3$, $B_2 = 1$)

$$\left(\frac{\delta}{P}\right)_{calc} = \frac{2 l^3}{3 E_f b t c^2} + \frac{l}{G_c^* b c}$$

t, c measured from micrographs (Table)

b, l from beam bending tests

E_f, G_c^* need to estimate

E_f - can be estimated from $E_{rib} V_{rib}$ in face (neglect contrib of square cells in face).

- ribs - sclerenchyma
- previous studies - sclerenchyma from grass leaf fibers $E_{scler} = 2\text{-}23\,GPa$
- tensile tests on iris leaves $E_{rib} = 21\,GPa$
- volume fraction of ribs in the faces is 0.39

$$E_f = 0.39\, E_{rib} = 8.2\,GPa$$

G_c^* - assume tissue fresh (E parenchyma constant at high/normal turgor pressure)

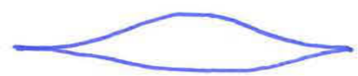
- data for $E_{parenchyma} = 0.5 - 6\,MPa$
- take $E_{parenchyma} \approx 4\,MPa$
- $G_c^* \sim \frac{1}{2} E_{parenchyma} = 2\,MPa$

Parenchyma Properties

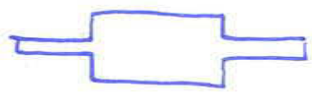
Plant material	Young's modulus, E^* or shear modulus, G^* (MPa)	Compressive strength, σ^*_{comp} (MPa)	Reference
Apple	E^* = 0.31-3.46	0.66	Oye et al., 2007
Apple	E^* = 2.8-5.8	0.25-0.37	Lin & Pitt, 1986
Apple	G^* = 1-6		Vincent, 1989
Potato	E^* = 3.6	1.3	Lin & Pitt, 1986
Potato	E^* = 3.5		Scanlon et al, 1996
Potato	E^* = 5.5	0.27	Hiller & Jeronimides, 1996
Potato	G^* = 0.5-1		Scanlon et al., 1996; 1998
Carrot	E^* = 2-14		Georget et al., 2003

Data for fresh, wet tissue, at normal turgor.

- using sandwich beam theory, can estimate P/δ (Table)
- calculation complicated by irregular thickness of core across section:



- rough attempt to account for this by dividing cross-section into sub-units



- found calculated P/δ overestimated measured P/δ by 16-83%
- agreement ok for the various approximations + estimates made.

Strength of the leaf

faces: $\sigma_{yf} = ?$

- previous tests on tensile strength of grass leaves found

$$\sigma_f \text{ (in MPa)} = 1.44\,(V_{sclerenchyma} \times 100) + 1.53$$

↑ vol. fraction

- in this, ribs (assume all sclerenchymal are 80% dense + make up 40% of face

$$\sigma_{yf} = 1.44\,(0.8 \times 0.4 \times 100) + 1.53 = 47\text{ MPa}$$

Iris Sandwich Analysis

Table 6.2 Beam bending results

Specimen	**1**	**2**	**3**	**4**
Measured beam stiffness, P/δ (N/mm)	0.66	0.54	0.41	0.25
Beam length, l (mm)	35	35	35	35
Face thickness, f (mm)	0.03	0.03	0.03	0.03
Maximum core thickness, c (mm)	4.63	3.31	2.49	1.51
Width, b (mm)	18	18	18	18
Flexural rigidity, D (Nm^2)	0.027	0.016	0.0096	0.0051
Bending compliance, $(\delta/P)_b$ (m/N)	5.29×10^{-4}	8.98×10^{-4}	1.49×10^{-3}	2.83×10^{-3}
Shear compliance, $(\delta/P)_s$ (m/N)	2.99×10^{-4}	3.83×10^{-4}	4.83×10^{-4}	6.3×10^{-4}
Calculated beam stiffness, P/δ (N/mm)	1.21	0.78	0.51	0.29
Calculated/measured beam stiffness	1.83	1.44	1.24	1.16

core : $\tau_c^* = ?$

- literature $\sigma^*_{tension} \approx 0.4$ MPa (parenchyma)
- expect $\tau_c^* \sim \frac{1}{2}\sigma^*_{tension} \sim 0.2$ MPa

- calculate strength of iris leaf in wind - cantilever, uniformly distributed load ($B_3 = 2$ $B_4 = 1$)
- calculate loads at base of leaf (M_{max}) : $t \sim 0.03$ mm $l \sim 600$ mm

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face yielding: $$\frac{P_{fy}}{bc} = B_3\, \sigma_{yf} \left(\frac{t}{l}\right) = (2)\,(47\text{ MPa}) \left(\frac{0.03}{600}\right) = 4.7 \text{ kPa}$$

face wrinkling: $$\frac{P_{fw}}{bc} = 0.57\, B_3\, E_f^{1/3}\, E_c^{*\,2/3} \left(t/l\right) = (0.57)\,(2)\,(8.2\times10^9)^{1/3}\,(4\times10^6)^{2/3}\left(\frac{0.03}{600}\right)$$
$$= 2.8 \text{ kPa}$$

core shear: $$\frac{P_{cs}}{bc} = B_4\, \tau_c^* = (1)\,(0.2\text{ MPa}) = 200 \text{ kPa}$$

- expect leaf failure by face wrinkling

- are iris leaves optimized?
  - $\delta_s / \delta_b = 0.22 - 0.57$ in specimens tested
  - in minimum weight design for given stiffness $\delta_s / \delta_b = 2$
  - but leaves have several functions beyond mechanical support:
    - photosynthesis, requires large surface area
    - fluid transport
  - difficult to quantify relative importance of each function to plant
  - engineering optimization not possible

Additional examples of sandwich structures in nature:

- marine "leaves" seaweed Durvillaea antarctica: fronds 12 m long honeycomb core
- bird skulls
  - if inner + outer face concentric - trabeculae oriented ⊥ cortical shell
  - if " " " not " - " foam-like
  - larger birds have multiple sandwiches
    - since size of trabeculae relatively constant, this may allow larger core thickness
  - owl skull - asymmetry - improves hearing

Comparison of optimized sandwich plate with solid plate of same stiffness

- consider circular plate, radius R, simply supported around circumference, subject to a uniformly distributed load q $[N/m^2]$
- central plate deflection is w
- if sandwich is optimized (based on analysis in book p 384)

$$\frac{\text{Mass}}{\pi R^2} = 1.49 \left(\frac{qR}{wE_s}\right)^{3/5} \rho_s R$$

(foam core)

- equivalent solid plate:

$$\frac{\text{mass}}{\pi R^2} = 0.89 \left(\frac{qR}{wE_s}\right)^{1/3} \rho_s R$$

- taking the ratio:

$$\frac{M_{sandwich}}{M_{solid}} = 1.67 \left(\frac{qR}{wE_s}\right)^{0.27}$$

- consider bone sandwich, "foamed" trabecular core; $R = 100$ mm $\quad P = 500$ N $(= q\pi R^2)$ $\quad w = 1$ mm $\quad E_s = 18$ GPa

$$\frac{qR}{wE_s} = 10^{-4} \qquad \frac{M_{sandwich}}{M_{solid}} = 14\%$$

⇒ optimized bone sandwich would be 14% weight of solid cortical panel

Additional examples (cont'd)

- cattle fish bone (not a fish - a mollusk ; not bone - $CaCO_3$)
- horseshoe crab shell.
- tortoise shell (Galapagos)

Durvillaea antarctica (New Zealand Seakelp)

Largest intertidal seaweed
Fronds up to 12m long
Fronds have gas-filled honeycomb-like core that provides buoyancy as well as flexural rigidity, maximizing surface area exposed to sunlight

http://en.wikipedia.org/wiki/File:Dried_bull_kelp_(Durvillaea_antarctica)_with_cross-section_showing_honeycomb_structure_IMG_102_1239.JPG

Bird Skulls

Alison Curtis

Photo of owl imprint in the snow removed due to copyright restrictions.

No footprints in the snow from mouse or vole; animal was under the snow
http://www.twincitiesnaturalist.com/2010/01/barred-owl-hunting-in-snow.html

Photo removed due to copyright restrictions. See Summit Post.
http://www.summitpost.org/disappearing-rabbit-trick/185785/c-186336

Rabbit tracks in snow
http://www.myconfinedspace.com/2006/12/21/owl-snowprint/

Cuttlefish bone
Mollusc shell ($CaCO_3$)

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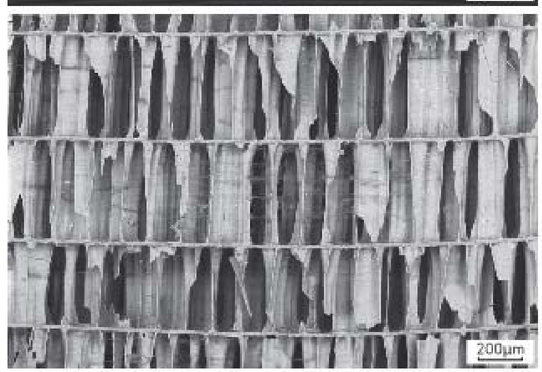
Source: Wikimedia Commons.



Horseshoe Crab Shell

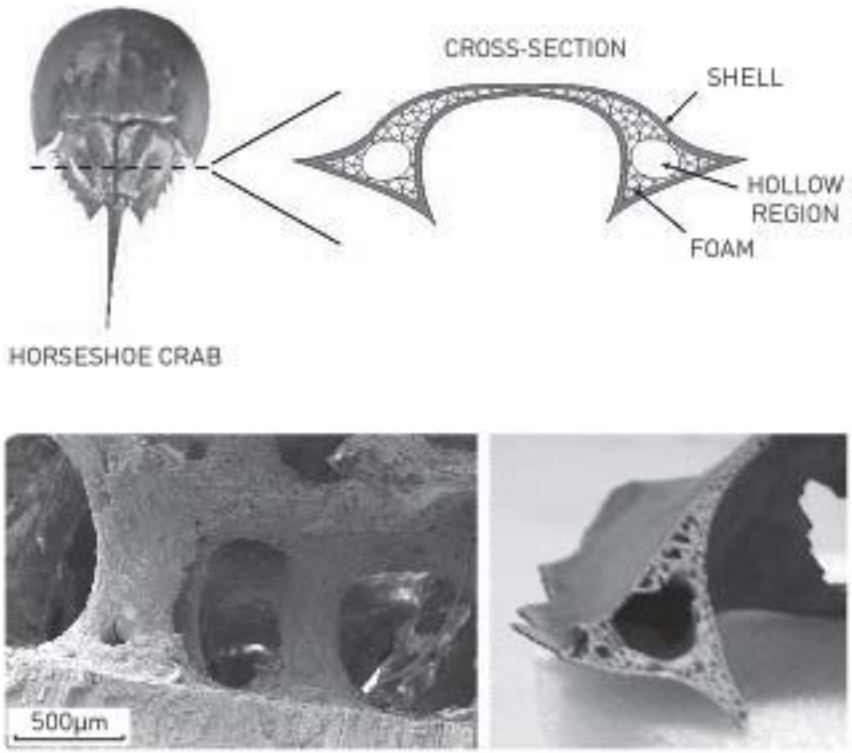


Figure 148: M. A. Meyers, P. -Y. Chen, et al. *Progress in Materials Science* 53 (2008): 1–206.

http://www.sciencedirect.com/science/article/pii/S0079642507000254

Meyers et al., 2008

Galapagos Tortoise Shell

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2014