Various Formulations of the Second Law of Thermodynamics

The Kelvin-Planck statement:

It is impossible to construct a process that, operating in a cycle, will produce no effect other than extraction of heat from a reservoir at one temperature and the performance of an equivalent amount of work.

The Clausius statement:

It is impossible to have a cyclic process whose sole result is a heat flow from a cooler body to a warmer body.

It is relatively easy to show that the Kelvin statement leads to the Clausius statement and vice versa. The relation with the next formulation may be a bit less obvious.

The Carathéodory statement:

Arbitrary close to any given thermodynamic state there exists states which cannot be reached from it solely by means of adiabatic processes.

These statements have mainly historical significance. We will add to these statements the one we will use:

There exists a STATE function S of the extensive variables for which holds: (for closed systems)

$$\mathrm{d}S \ge \frac{\delta Q}{T}$$

For a reversible process the equality sign holds:

dS
$$\frac{\delta Q}{T}$$