3.22 Mechanical Properties of Materials
Spring 2008

Problem Set #6

Due: Friday, May 9 by 5:00 PM

1. Describe thoroughly yet concisely three (general) toughening mechanisms in materials. Be sure to include an example of each, and not to confuse (yield) strengthening with toughening. For one mechanism, show a fractograph and briefly note features in the fracture surface that illustrate how the mechanism works.

   Solution: The theme of toughening mechanisms is energy dissipation. The more energy the material can dissipate through other means besides crack growth, the tougher the material can be. Examples of this include phase transformations, crack deflection, and second phase pull-out.

   - *Phase transformations – The energy put into the material by an applied stress/strain is dissipated when the material at the crack tip (where the stress concentration is highest) undergoes a phase transformation.*
   - *Crack deflection – If the crack is deflected around tougher precipitates/particles (in small volume fraction) or along grain boundaries (intergranular cracking as opposed to transgranular cracking), the crack must travel a longer distance, which requires more energy.*
   - *Fiber pullout – This toughening mechanism operates in composite materials where the reinforcement phase is in fiber form and the fibers are tougher than the fiber/matrix interface. In this scenario, a propagating crack will be arrested at the fiber. Eventually enough energy will be applied such that the fiber will be pulled out of the matrix.*

   See also the crack bridging concept summarized in MIT Server *Forum/Questions, and the mechanisms named slightly differently in the context of bone fracture and fatigue failure discussed in Lecture 20.*

2. Lowhaphandu and Lewandowski [Scripta Mater. 38, 1811(1998)] studied the effect of crack tip radius on the stress intensity factor at failure for a specific material, as summarized in the graph below. (Note that they call crack tip radius "notch root radius" and stress intensity factor at failure "fracture toughness".)

   (a) Using Inglis's analysis of stress at the tip of an elliptical hole, calculate the stress at the crack tip (as a function of applied stress) for the largest and smallest crack tip radius considered. (This is an single edge-crack with initial crack length a of 25 mm.)

   Solution: Inglis's analysis for the stress σ at the tip of a crack with length a and radius of curvature R is

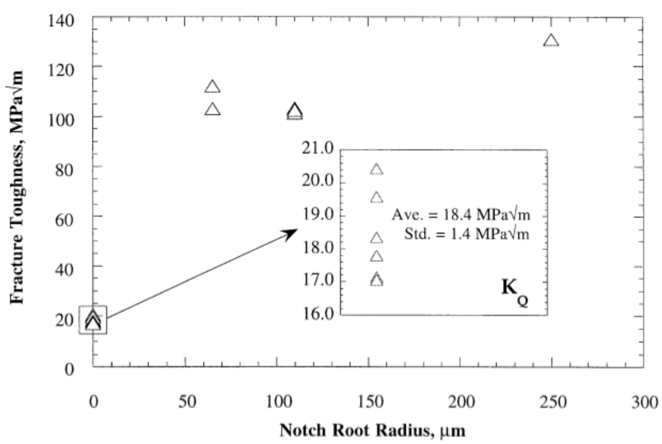
   $$\sigma = \sigma_\circ \left(1 + 2\sqrt{\frac{a}{R}}\right)$$

   where $\sigma_\circ$ is the applied (global or far-field) stress. Using this relation with $a = 25$ mm, we find that $\sigma = 21\sigma_\circ$ for $R = 250\ \mu\text{m}$ and $\sigma = 20001\sigma_\circ$ for $R = 2.5$ Å. (Note that I assumed an "atomically sharp" crack tip radius for the second example, even though it is known that the crack is NOT atomistically sharp. Anything $\leq 1\ \mu\text{m}$ is acceptable, however.)

(b) Are your results from part (a) consistent with the results in the graph below (i.e., consistent with the fact that at the largest crack tip radius we measure has the largest stress intensity factor)? Explain why they are or are not consistent.

Solution: Yes these results are consistent. These results suggest that a sharper crack tip acts as a stronger stress concentrator than a more blunt crack tip. Therefore it would will take less stress to reach the "critical stress intensity" needed to fracture the material.

You should also consider whether this answer will be dinfferent when you treat the stress concentration as the prefactor magnifying the applied stress in the Inglis solution. This prefactor can be called the stress concentration or stress intensity factor, but mean different things depending on whether the person is assuming an atomistically sharp crack (linear elastic solutions that gave us K_I) or not (like the prefactor of the hole in a plate discussed in PS5.)



3. For a single material (which follows Paris Law steady-state crack propagation with $m \neq 2$), you have tested three components with initial crack sizes $a_\circ$ which failed with final crack sizes a_f (listed in the table below).

   (a) Assume the fatigue conditions of all components are uniaxial loading with $R = -1$ at a maximum stress of 100 MPa and a frequency f of 10 Hz. Draw $\sigma(t)$ for two complete loading cycles, and also draw the representative volume elements and associated stress states at the minimum and maximum $\sigma(t)$.

   Solution:

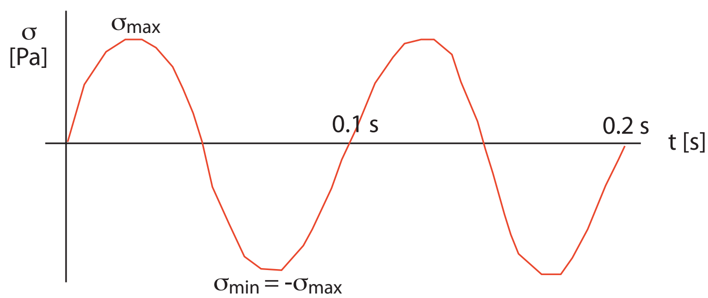


Figure 1: Uniaxial fatigue loading to σ_{max} = 100 MPa at f = 10 Hz.

   (b) Compare the differences in fatigue lifetimes for the three components. Explain your findings in terms of crack growth rates. From this, what do you infer about how growth of cracks should be monitored in a material component put into application?

Component	$a_\circ$	a_f
A	2 mm	10 mm
B	0.5 mm	10 mm
C	2 mm	40 mm

Solution: By integrating the Paris Law for steady-state fatigue crack growth, we find that the cyclic lifetime of a material N_f is proportional to the initial crack size $a_\circ$ and final crack size a_f by

$$N_f \propto \left(\frac{1}{a_\circ^{(m-1)}} - \frac{1}{a_f^{(m-1)}} \right) = M.$$

Therefore we can compute a multiplier M for each component. I assumed $m = 2.9$ (the value for a common steel) and found

Component	$a_\circ$	a_f	*M*
A	*2 mm*	*10 mm*	*400*
B	*0.5 mm*	*10 mm*	*1900*
C	*2 mm*	*40 mm*	*475*

Assuming the stress range is the same for each component, from this we see from the case where the initial crack lengths are different but the final crack lengths are the same (components A & B), an initial crack four times smaller results in a 375% *increase in cyclic lifetime. However in the case where the initial crack lengths are the same but final crack lengths are different (components A & C), a final crack length 4 times longer results only in a* 19% *increase in cyclic lifetime. These results show that crack growth is faster for larger cracks and that cyclic lifetime is most sensitive to the initial crack size. Mathematically, one can see this in the natural log dependence of initial/final crack sizes on* N_f *for the* $m = 2$ *solution of Paris' Law. Therefore, when a material experiences fatigue loading in an application and a crack is visible to the eye, it must be closely monitored because most of the cycles to failure will have been used to grow the crack to a visible size.*

(c) Comment on the average stress applied to this material, and why this is not counter to the reality that fatigue failure still occurs in all of these components.

Solution: The average stress for $R = -1$ *is necessarily zero, since you are cycling the material between equal/opposite tensile and compressive stress. However, a time-averaged stress of zero still results in fatigue failure because the crack will open a portion* da *for every incremental cylce* dN *if the crack growth conditions are within the Paris law, steady-state crack growth regime. For every maximum tensile load cycle achieved, the crack will be under a driving force to open the crack faces. Also, although it may seem counterintuitive, it has been shown experimentally that cracks can also propagate under compressive loading because the plastic zone ahead of the crack tip can be sufficiently strained to attain the compressive fracture stress.*

4. With your special topic group, please prepare slides for a short presentation of your topic to the class, to be delivered on Tuesday, May 13. A template will be posted for your convenience on the wiki. Be as creative as you like, but please adhere to the guidelines to provide uniformity of details and keep to our time constraints.

   Reminder that you will need to respond to 2 of 3 wiki questions on the last quiz (i.e., 2 that are not your own special topic). The questions will be based on content delivered in these presentations and related lecture/pset material from the semester. However, you will likely find it helpful and enjoyable to peruse the wiki pages of your colleagues to see more than they will have time to share in class.