Problem Set 6

Late homework policy. Late work will be accepted only with a medical note or for another Institute-approved reason.

Cooperation policy. You are encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding. You may not copy another student's solutions. And you should not refer to notes from a study group while writing up your solutions (if you need to refer to notes from a study group, it isn't really "your own understanding").

Part I. These problems are mostly from the textbook and reinforce the basic techniques. Occasionally the solution to a problem will be in the back of the textbook. In that case, you should work the problem first and only use the solution to check your answer.

Part II. These problems are not taken from the textbook. They are more difficult and are worth more points. When you are asked to "show" some fact, you are not expected to write a "rigorous solution" in the mathematician's sense, nor a "textbook solution". However, you should write a clear argument, using English words and complete sentences, that would convince a typical Calculus student. (Run your argument by a classmate; this is a good way to see if your argument is reasonable.) Also, for the grader's sake, try to keep your answers as short as possible (but don't leave out *important* steps).

Part I(20 points)

| (a) | (2 points) | p. 235, | Section 7.4, | Problem 18(a) | Ignore the introductory |
|-----|-------------|---------|------------------|-----------------|--------------------------|
| (b) | (2 points) | p. 235, | Section 7.4, | Problem $18(b)$ | physics note/"problem" |
| (c) | (2 points) | p. 591, | Section 17.1, | Problem $12(a)$ | |
| (d) | (2 points) | p. 591, | Section 17.1, | Problem $12(b)$ | |
| (e) | (2 points) | p. 240, | Section 7.5 , | Problem 10 | |
| (f) | (2 points) | p. 244, | Section 7.6, | Problem 7 | |
| (g) | (2 points) | p. 563, | Section 16.1, | Problem 7 | |
| (h) | (2 points) | p. 567, | Section 16.2 , | Problem 1 | 12 and 15 are also fun |
| (i) | (2 points) | p. 579, | Section 16.4, | Problem 14 | |
| (j) | (2 points) | p. 582, | Section 16.5 , | Problem 7 | |

Part II(30 points)

Problem 1(10 points) Solve Problem 41 on p. 256. In other words, compute the volume of a torus using the shell method.

Problem 2(20 points) The *three-leaved rose* is the polar curve,

$$r = \cos(3\theta).$$

(a)(5 points) Sketch the polar curve for $-\pi/2 \le \theta \le \pi/2$. In a separate box, list the angles where r is maximum, the angles where r is 0, and the slope of the curve at each point where r is 0.

(b)(5 points) Compute the area of the leaf of the rose that contains the point ($\theta = 0, r = 1$).

(c)(5 points) Read Problem 11, §9.1, p. 300. You may use the solution from the back of the book; the problem is very similar to Problem 12, which was Part I, (h) from Problem Set 2. Using the solution, find an implicit equation for the Cartesian form of the curve, e.g. some polynomial in x and y equals 0.

(d)(5 points) Using your solution to (c), write down an integral of the form,

$$2\int_{x=0}^{x=1}f(x)dx,$$

for the area of the leaf in (b). Do not attempt to solve this integral; you already computed its value.