Introduction to Curve Sketching

Goal: To draw the graph of f using information about whether f' and f'' are positive or negative. We want the graph to be qualitatively correct, but not necessarily to scale.

WARNING: Don't abandon your precalculus skills and common sense — you already know a lot about graphing functions; calculus just fills in the gaps.

The first principle is that if f' is positive, then f is increasing. In other words, if the tangent line is pointing up then the function is going up too. Similarly, if the derivative of f is negative then f is decreasing.

The second principle is just a second order effect of the same type. If f'' is positive, that means that f' is increasing. This is just the first principle applied to the second derivative; f'' is the derivative of f'.

Figure 1 shows the graph of a function for which the second derivative is positive. You can see from the tangent lines sketched on the graph that their slopes increase from negative on the left to positive on the right. We say that curves with this shape are *concave up*.

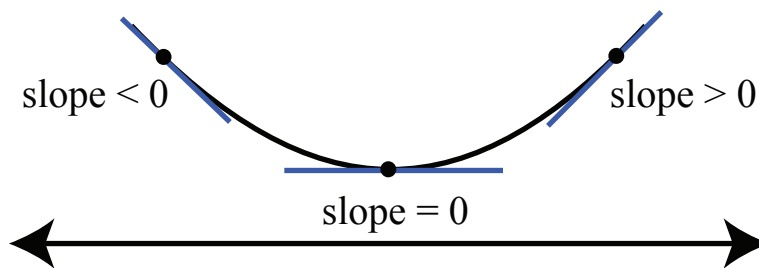Similarly, if $f'' < 0$ then f' is decreasing and the graph of f is *concave down*.

Figure 1: f is concave-up. The slope increases from negative to positive as x increases.

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