## Choosing a Technique

**Question:** How will we know which method to use on the exam?

**Answer:** You'll need all three methods of Riemann sums, trapezoidal rule and Simpson's rule.

Volumes of revolution always depend on a two dimensional diagram which you should be able to figure out. (There won't be any volumes as complicated as the surface $e^{-r^2}$ on the exam.)

Once you've got the two dimensional diagram for a volume or area calculation, you'll have a choice between integrating with respect to $x$ or $y$. If there's some choice that seems reasonable but makes the problem very hard, you might get a hint suggesting you use the washer method, for example. If, on the other hand, there's one choice that's reasonable and another that's obviously very difficult, you may be left to figure out which is the right choice on your own.

For example, suppose you're looking at the volume formed by rotating the region $0 < y < x - x^3$ shown in Figure 1 about the $y$-axis.

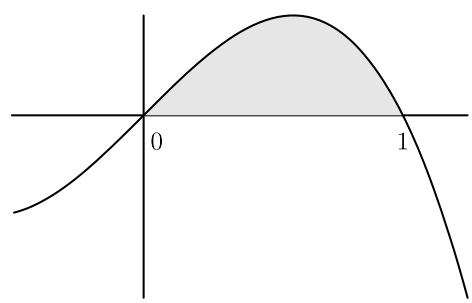


Figure 1: $0 < y < x^3 - x$.

We have to choose whether to integrate with respect to $x$ or with respect to $y$. If we choose to integrate with respect to $x$, we'll be adding up volumes of cylindrical shells with thickness $dx$ and height $x - x^3$.

$$\int_0^1 2\pi x(x - x^3)\, dx$$

When we reach this point, we know we've made a good choice because this is an easy integral to calculate.

If, on the other hand, we decided to integrate with respect to $y$, we'd be adding up volumes of "washers" – disks with circular holes in their centers. (See Figure 2.) Using the washer method, we get the following formula for the volume:

$$\int \pi(x_2^2 - x_1^2)\, dy$$

We can already see that this is more complicated than the other, but let's discuss another step of the calculation to see how much more complicated. To compute this integral, we'll need to solve $y = x - x^3$ for $x$ in terms of $y$. It's not easy to

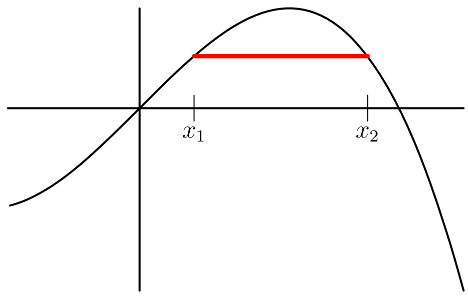


Figure 2: Rotate this rectangle about the $y$-axis to get a washer shaped volume.

solve $x^3 - x + y = 0$; you won't be able to do it during the test. So integrating with respect to $y$ is the wrong choice for this problem.