Riemann Sum Practice

Use a Riemann sum with $n = 6$ subdivisions to estimate the value of $\int_0^2 (3x + 2)\,dx$.

Solution

This solution was calculated using the *left Riemann sum*, in which $c_i = x_{i-1}$ is the left endpoint of each of the subintervals of $[a, b]$. To denote the heights of the rectangles we let $y_i = f(x_i)$, and so obtain the following expression for the left Riemann sum:

$$f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x = (y_0 + y_1 + \ldots + y_{n-1})\Delta x.$$

In our example, $n = 6$, $a = 0$, $b = 2$, $\Delta x = \dfrac{b-a}{n} = \dfrac{1}{3}$, and the values y_i correspond to the height of the graph of $y = 3x + 2$ at the left edge of each interval, as illustrated in Figure 1.

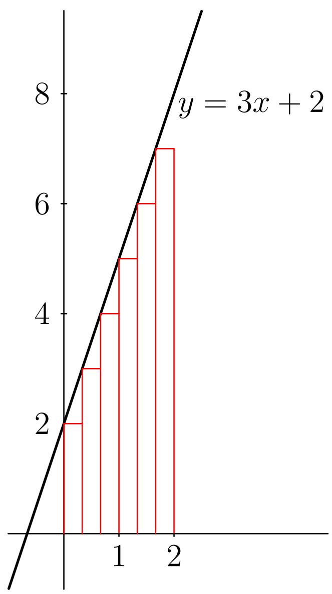

Figure 1: Rectangles used to compute the Riemann sum.

We could compute $x_i = a + i\Delta x = \frac{i}{3}$ and so $y_i = 3(x_i) + 2 = i + 2$ and $c_i = \frac{i-1}{3}$, or we could simply mark off the left endpoints $0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{1}{3}$ and $\frac{5}{3}$ and then read the heights of the rectangles from the graph. In either case, our formula for the left Riemann sum tells us that the area under the graph of $3x+2$ between $a = 0$ and $b = 2$ is approximately:

$$(y_0 + y_1 + \ldots + y_{n-1})\Delta x = (2 + 3 + 4 + 5 + 6 + 7) \cdot \frac{1}{3} = 9.$$

Because $\int_0^2 (3x+2)\,dx$ is the area of a trapezoid with width 2 and sides of height 2 and 8, we can easily check our work:

$$\int_0^2 (3x+2)\,dx = 2 \cdot \frac{2+8}{2} = 10.$$

From the figure we see that the left Riemann sum slightly underestimates the area, so our answer of 9 is probably correct.