Trapezoid Rule Approximation of $\int_1^2 \frac{dx}{x}$

Continuing our discussion of numerical integration, we'll look at:

$$\int_1^2 \frac{dx}{x}$$

Of course we already know:

$$\begin{aligned} \int_1^2 \frac{dx}{x} &= \ln x|_1^2 \\ &= \ln 2 - \ln 1 \\ &= \ln 2 \end{aligned}$$

We can use a calculator to find that this value is approximately 0.693147.

Numerical methods allow us to estimate integrals with accuracy about equal to our accuracy in estimating the integrand. We can approximate the value of $1/x$ pretty well, so we can get a pretty accurate estimate of the value of $\ln 2$. To make life even easier for ourselves, we'll do a very simple case of the approximation — we'll only use two intervals.

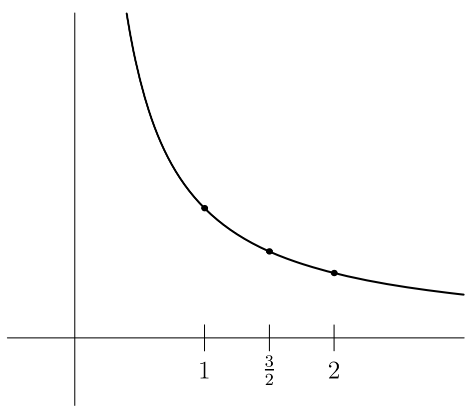

Figure 1: Two intervals; three points on the hyperbola.

We can't expect to get a very good approximation of $\ln 2$ using only two intervals. With only two intervals, we're making estimates of the area under a hyperbola based on only three points (see Figure 1).

Trapezoidal Rule

The trapezoidal rule gives us the following formula for the area under the curve:

$$\text{Area} \approx \Delta x \left(\frac{1}{2} y_0 + y_1 + \frac{1}{2} y_2 \right).$$

In this case $\Delta x = \frac{b-a}{n} = \frac{1}{2}$ because $b = 2$, $a = 1$ and $n = 2$. By evaluating $y_i = f(x_i) = \dfrac{1}{1 + \frac{1}{2}i}$ and plugging these values in, we get:

$$\text{Area} \approx \frac{1}{2}\left(\frac{1}{2} \cdot 1 + \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{2}\right)$$

You wouldn't have to perform this addition on an exam, but if there are only two terms you should add them together. When we do put these numbers into a calculator, we find that the trapezoidal rule gives an estimate of about 0.708 for the area of this region; that's pretty close under the circumstances!