Welcome back to recitation. In this video, I want us to practice doing integration again. And so what we're going to do is two problems. One is a definite integral, one is indefinite. So this integral, we're going to have, find the value of the integral minus pi over 4 to pi over 4 of secant cubed u du, and then this one, I just want you to actually just find an antiderivative, then, for 1 over 2 sine u cosine u.

So why don't you take a while to work on this, pause the video, and then when you're ready, restart the video, and I will come back and show you how I did it.

OK. Welcome back. So again, we're going to work on finding, in this one, a specific number, and in this one, an antiderivative. So we'll start off with the integral from minus pi over 4 to pi over 4 of secant cubed u du. And the first thing I'm going to do, is I'm going to drop the bounds and just find the function I need, and then I'll bring the bounds back in. I don't want to write the bounds down at every step. I'm not doing any substitution, I don't think. So I don't need to worry about if I'm changing the bounds or not. So I'll keep the bounds the same, but I won't write them down again until near the end.

All right. So let's look at this integral. So the easiest way, I think, to start an integral like this, is to split it up into secant u and then secant squared u. Maybe there are some other ways you did it, but when I was looking at this problem, the way I saw was to start off and write this as secant u times 1-- oops, let me make sure, yes-- times 1 plus tan squared u du. Because secant squared u is equal to 1 plus tangent squared u.

And what does it do for us? Well, it partially answers the question, but not completely, we'll see. So what it does first, is it gives us something we can find pretty easily. The first one, we just get the integral of secant u du. We know that one. The second one is a little harder, because we get the integral of secant u tangent squared u du.

And this has promise, but it's not going to work for us right away. Because, you know, the derivative of tangent is secant squared, and the derivative of secant is secant tangent. So a substitution won't work in this case, because neither one of these functions is a derivative of the other one. If I even move off a tangent, and I have secant tangent times another tangent, it's not the right derivative. A tangent's derivative is secant squared. So my main point here is that a u-substitution doesn't work, or the substitution strategy doesn't work.

So what we're going to do, is we're actually going to take this part, and make it into an integration by parts problem. So this is going to be, I'm going to stop writing equals signs, I'm going to figure out the integral of this one. So this is a little sidebar, and I'm going to look at the integral of secant u tangent squared u.

And the way I'm going to write that, is I'm going to write that as secant u tangent u will be one function I want. We'll call that w-- I guess, v prime. We can use v prime. And then the usual thing we call u, we'll call w. So tangent u we'll call w. And I want to write down, I want to explain to you why I'm picking these, and sort of where this is going to get us.

So this is an easy thing to take an antiderivative of. Right? I can take this. I know v is going to be secant u. Because the derivative of secant u is secant u tangent u. And then w is tangent u, w prime is secant squared u. So if you think about, what does an integration by parts have? It's going to have v*w minus the integral of v w prime. That's a lot to take in. But the point is that that integral is going to have the antiderivative of this, which is secant, and the derivative of this, which is secant squared. So it's going to have a secant cubed. Which might seem weird, because now we're getting back to what we started with. But the sign on the secant cubed is going to be opposite.

Again, this is a lot of talking. But let's figure out now. I just want to show you where we're headed with this, why I picked the things I did.

So as I mentioned-- let me just write these down-- secant u is equal to v, and secant squared u is equal to w prime. Sorry if that looks a little weird, but that's a u.

OK. So now let's write this with an integration by parts. I get v*w, which is secant u tangent u minus the integral of v*dw, which is secant cubed u du.

So this is what I was trying to show you we were anticipating. So when I put this all together, I replace this integral by these two things. And what's the point there? Notice what I have. If I actually look at the pieces, I have, up here, I have a secant cubed u. It's going to equal secant u plus this-- I have to evaluate that at the bounds-- minus this. So I have the same thing on this side that I had on the other side, but with a minus sign. So if I add it to the other side, I'm going to get two of them.

So this was sort of where we're headed. Now let's put it all together. Let's take it back. So this stuff here is that, right? That's what we did. I'm going to write just the important stuff right here. I've got the integral of secant cubed u du is equal to secant u tangent u. Plus that secant-- I forgot to put that one in, so let me write in that one. Plus the integral of secant u du minus secant cubed u du. The integral of secant cubed u du. OK?

And now I'm going to work some magic. I'm actually going to erase something and move it to the other side. So let me sneak an eraser off screen. I'm going to add this to the other side. And what's going to happen? I come over here and I get two of them. Right? There was one on this side, with a minus sign. I added it, and now I have two of them. And now what's the magic? Well, I just divide everything by 2. And so this is going to be over 2, and this is going to be over 2.

And now I know what an antiderivative is. Notice I haven't put in the plus c, because I'm about to put in some bounds. All right? And by the way, I know an antiderivative of secant u. So we'll get to that in one second. But hopefully everyone follows. I had an integral of secant cubed u on this side. I had a minus integral of secant cubed u over here. I added it to the other side. It gave me two of them, so then I just divided by 2. All right?

And now let me just explicitly write down the last thing we need. We still need this one. And this is going to be 1/2 natural log absolute value secant u plus tangent u.

OK. So now we just have to evaluate everywhere and we're done. So now we have to evaluate all of this. Remember, I said I left out the bounds. The bounds are pi over 4, minus pi over 4; pi over 4 minus pi over 4.

All right. So I'm going to give myself a little cheat sheet up here, and then I'm going to write down the numbers I get here. So my cheat sheet is to remind myself that secant of plus or minus pi over 4 is equal to square root 2. Let me just make sure that's right. Cosine pi over 4 is 1 over square root 2. Cosine is even, so plus or minus pi over 4 will be the same. Secant is 1 over that, so I'm good.

Tangent of pi over 4. Well, tangent is odd, so I should say plus or minus here, tangent is odd, so it's going to be, they're going to have two different signs. Tangent pi over 4 is sine pi over 4 over cosine pi over 4. It's the same value there. So it's 1. So tangent plus or minus pi over 4 is plus or minus 1.

So that's what we're working with. So now let's start plugging in. Secant pi over 4 tangent pi over 4 gives me root 2 times 1 over 2. So I get root 2 over 2, is the first thing. So I have to write-- this equals is from here. So I have root 2 over 2. And then secant minus pi over 4 is again root 2, tangent minus pi over 4 is minus 1. So I have minus, I have a negative here, and then I have a 1 here, or a root 2 here, negative 1 over 2. So I get another negative, so I have a plus. So one negative came from, I was using the lower bound, and one negative came from the tangent. That gave me a plus.

And then I have plus 1/2. Well, again. Natural log of secant pi over 4 is going to be-- so I'm going to have natural log of root 2 plus 1, and I'm going to have a natural log of root 2 minus 1. And I'm going to have a negative in between them. So I'm going to work a little magic here. It's going to be natural log of 2 plus 1 over root 2 minus 1.

Now, just to point out, where did that come from? That came from putting in root 2 for both of the pi over 4's and minus pi over 4. Tangent pi over 4 was the plus 1. Tangent of negative pi over 4 was the minus 1. How do I get this division? I had natural log of something minus natural log of something else.

So in the end, I get root 2 plus 1/2 natural log absolute root 2 plus 1 over root 2 minus 1.

All right. That is part (a). So part (a), let me just remind you, what did we do? We had secant cubed u. We did a substitution for one of the, for secant squared. We got something we could deal with, and something that didn't look so promising, but once we did an integration by parts, we were back to what we started with, with a different sign. So we moved it to the other side. We were basically able to solve for secant cubed u. So we got all the way through, and then we just had to evaluate.

So the big step was, once you were here, knowing an integration by parts actually would save you. That's sort of the hard thing to see. Takes a little while to see that one, maybe.

All right. So now the next one. If we come back here, we're trying to find an antiderivative of 1 over 2 sine u cosine u. And there may be some other ways to do this, but actually, this problem, part of the reason I wanted to do this problem, was I wanted to remind you that it's good to know some of the basic trigonometric identities, because it'll make your life a lot easier.

So this integral is actually the same as, is the integral of du over sine 2u. And the reason is, there's a trigonometric identity that says, 2 sine u cosine u is equal to sine 2u. So I wanted to give you a reason for why we know those, why we know some of those identities, and you end up in these situations. There might be other ways to handle this one, but the easiest, most direct way is if you do this. You change it so that (b) is actually the same thing as integral of du over sine 2u. So it's just a straight up double angle formula, you can call it if you need a fancy name.

And now what is this? Well, this is equal to the integral of-- what's 1 over sine, is going to be cosecant. Cosecant 2u du. And we know the antiderivative to cosecant u. We know that that's going to be negative natural log of cosecant u plus cotangent u. But the problem is, when there's a 2 there, what do we do? Well, just think about it as, if you had the antiderivative, you know by the chain rule, if you just put in two u's everywhere there was a u when you took the derivative, you would end up with an extra 2 in front. So you have to, basically you have to just put in 1/2 in front.

You could do a substitution to check this, but it's really straightforward to say, this antiderivative is equal to negative 1/2 natural log absolute cosecant 2u plus cotangent 2u.

Now that I have that written out, I'll just point out again, if there was no 2 here, the 1/2 wouldn't be here, and we'd just have cosecant u cotangent u inside here. But once we have to have the 2 to get a cosecant 2u in the end, we also need to divide by 2 to kill it off when we take a derivative. The chain rule would give us a 2 in front, so the 1/2 kills it off. So we don't end up with, you know, with this not here, the derivative of this is 2 cosecant 2u. So we divide by 2, then we get it. We get the right answer.

So this one-- you know, I'm not intentionally trying to trick you. I just want to point out that it's good to know some of these trigonometric identities. It makes solving these problems a lot easier to deal with. So the main point of this one was actually knowing the identity, in my mind. Maybe you found another way to do it. Probably it didn't take two lines, though. So if you found other way to do it, actually, it's good. It's creative. And I like that. But I was hoping to convince you that sometimes it's simple to know a few of these identities.

And that is where I will stop.