Example: $\int \ln x \, dx$

This looks intractable, but if we fit it into the form $\int uv' \, dx$, integration by parts makes the calculation relatively easy.

Here's the idea: if we let $u = \ln x$ then when we apply the formula for integration by parts we'll get an integral involving $u' = \frac{1}{x}$. The key element is that the derivative of $u = \ln x$ is easier to integrate than what we started with.

In order to fit the form $\int uv' \, dx$ we need a function v. If we choose $v = x$ then $v' = 1$ and:

$$\int \ln x \, dx = \int uv' \, dx.$$

The formula for integration by parts is:

$$\int uv' \, dx = uv - \int u'v \, dx.$$

So by plugging in $u = \ln x$ and $v = x$ we get:

$$\begin{aligned} \int \underbrace{\ln x}_{uv'} \, dx &= \underbrace{\ln x \cdot x}_{uv} - \int \underbrace{\frac{1}{x}}_{u'} \underbrace{x}_{v} \, dx \\ &= x \ln x - x + c \end{aligned}$$

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