Introduction to Improper Integrals

An *improper integral* of a function $f(x) > 0$ is:

$$\int_a^\infty f(x)\,dx = \lim_{N\to\infty} \int_a^N f(x)\,dx.$$

We say the improper integral *converges* if this limit exists and *diverges* otherwise.

Geometrically then the improper integral represents the total area under a curve stretching to infinity. If the integral $\int_a^\infty f(x)\,dx$ converges the total area under the curve is finite; otherwise it's infinite. (See Figure 1.)

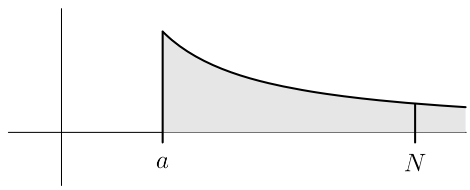

Figure 1: Infinite area under a curve.

How can an area that extends to infinity be finite? Obviously the area between a and N (i.e. $\int_a^N f(x)\,dx$) is finite. As N goes to infinity this quantity will either grow without bound or it will converge to some finite value. Our next step is to look at examples of each of these possibilities.