

TWELFTH HOMEWORK, PRACTICE PROBLEMS

1. Let $\vec{F}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the vector field given by

$$\vec{F}(x, y, z) = ay^2\hat{i} + 2y(x+z)\hat{j} + (by^2 + z^2)\hat{k}.$$

- (i) For which values of a and b is the vector field \vec{F} conservative?
- (ii) Find a function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $\vec{F} = \text{grad } f$, for these values.
- (iii) Find the equation of a surface S with the property that for every smooth oriented curve C lying on S ,

$$\int_C \vec{F} \cdot d\vec{s} = 0,$$

for these values.

2. Let S be the rectangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 2, 2)$ and $(0, 2, 2)$. Find the flux of the vector field $\vec{F}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, given by

$$\vec{F}(x, y, z) = y^2\hat{i} - z^2\hat{j} + x^2\hat{k},$$

through S in the direction of the unit normal vector \hat{n} , for which $\hat{n} \cdot \hat{k} > 0$.

3. Let $C_a(P)$ be the circle of radius a centered at P and oriented counter-clockwise. A smooth rotation free vector field \vec{F} is defined on the whole of \mathbb{R}^2 , except for the points $P_0 = (0, 0)$, $P_1 = (4, 0)$, and $P_3 = (8, 0)$, and

$$\int_{C_2(P_0)} \vec{F} \cdot d\vec{s} = -2, \quad \int_{C_6(P_0)} \vec{F} \cdot d\vec{s} = 1 \quad \text{and} \quad \int_{C_{10}(P_0)} \vec{F} \cdot d\vec{s} = 3.$$

Find the following line integrals.

(a)

$$\int_{C_1(P_1)} \vec{F} \cdot d\vec{s}.$$

(b)

$$\int_{C_1(P_2)} \vec{F} \cdot d\vec{s}.$$

(c)

$$\int_{C_6(P_2)} \vec{F} \cdot d\vec{s}.$$

4. (6.3.16)

5. (7.1.4)

6. (7.1.20)

7. (7.2.13)

9. (7.2.17)

10. (7.3.11)

11. (7.3.13)

12. (7.3.16)

13. (7.3.18)

14. (7.3.19)

Just for fun: Let ω be a k -form on \mathbb{R}^n . Show that

$$d(d\omega) = 0.$$

This basic fact about d is often expressed in the formula $d^2 = 0$.

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18.022 Calculus of Several Variables
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