EIGHTH HOMEWORK

Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (10 pts) (4.2.1)
2. (5 pts) (4.2.6)
3. (5 pts) (4.2.8)
4. (10 pts) (4.2.22)
5. (10 pts) (4.2.23)
6. (5 pts) (4.2.33)
7. (5 pts) (4.2.46(b))
8. (20 pts) We will show in this problem that amongst all boxes with surface area a, the volume is a maximum if and only if the box is a cube. Let

$$V\colon \mathbb{R}^3 \longrightarrow \mathbb{R},$$

be the function $V(x, y, z) = xyz$. Then we want to maximise V on the set A of all points where $x > 0$, $y > 0$, $z > 0$ and $2(xy + yz + zx) = a$.
(i) Show that there is a unique point $P \in A$ where V has a constrained critical point.
Let $K \subset A$ be the subset of points (x, y, z) where

$$x \geq \frac{\sqrt{a}}{3\sqrt{6}} \qquad y \geq \frac{\sqrt{a}}{3\sqrt{6}} \qquad \text{and} \qquad z \geq \frac{\sqrt{a}}{3\sqrt{6}}.$$

(ii) Show that if $Q \in A - K$ (so that Q is in A but not K) then $V(Q) < V(P)$.
(iii) Show that K is compact.
(iv) Show that V has a maximum on K at P.
(v) Show that V has a maximum on A at P.

8. (10 pts) (4.3.2)
9. (10 pts) (4.3.8)
10. (10 pts) (4.3.18)

Just for fun: What is the value of the limit:

$$\lim_{x \to 0} \frac{\sin(\tan x) - \tan(\sin x)}{\sin^{-1}(\tan^{-1}(x)) - \tan^{-1}(\sin^{-1}(x))}.$$