Computing 3x3 determinants

1. a) Compute $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$. b) Compute $\begin{vmatrix} 2 & 1 & -5 \\ 0 & 0 & 4 \\ 3 & 1 & 2 \end{vmatrix}$.

<u>Answer:</u> a) Using Laplace expansion along the first row (and remembering to put minus signs at the appropriate place) we get

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\cdot \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\cdot \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\cdot \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = (45-48) - 2(36-42) + 3(32-35) = -3+12-9 = 0.$$

To show everything we wrote out the arithmetic in detail. You would not need to show all this. It's usually easier to do the simple arithmetic in your head –but, never be afraid of writing out the details. The shorter answer would look something like

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = -3 + 12 - 9 = 0.$$

b) Because of the zeros we'll use Laplace expansion along the second row. (The signs are $- + -$, but the first two terms are 0.)

$$\begin{vmatrix} 2 & 1 & -5 \\ 0 & 0 & 4 \\ 3 & 1 & 2 \end{vmatrix} = -4 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -4(2-3) = 4.$$

2. Compute $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 6 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 5 & 1 & 2 \end{vmatrix}$

<u>Answer:</u> In principle a 4x4 matrix requires us to compute four 3x3 determinants. Here we can expand along the second row so we'll only have one non-zero term. The signs for the second row are $- + - +$.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 6 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 5 & 1 & 2 \end{vmatrix} = -6 \begin{vmatrix} 1 & 2 & 4 \\ 1 & 0 & 0 \\ 1 & 5 & 2 \end{vmatrix}$$

This 3x3 determinant can be expanded along its second row.

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & 0 & 0 \\ 1 & 5 & 2 \end{vmatrix} = -(-16) = 16.$$

So the original determinant is $-6 \cdot 16 = -96$.