Cross product

1. a) Compute $\langle 1,3,1\rangle \times \langle 2,-1,5\rangle$.

b) Compute $(\mathbf{i}+2\mathbf{j}) \times (2\mathbf{i}-3\mathbf{j})$.

<u>**Answer:**</u> a) We use the determinant method:

$$\langle 1,3,1\rangle \times \langle 2,-1,5\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 1 \\ 2 & -1 & 5 \end{vmatrix} = \mathbf{i}(16) - \mathbf{j}(3) + k(-7) = \langle 16,-3,-7\rangle$$

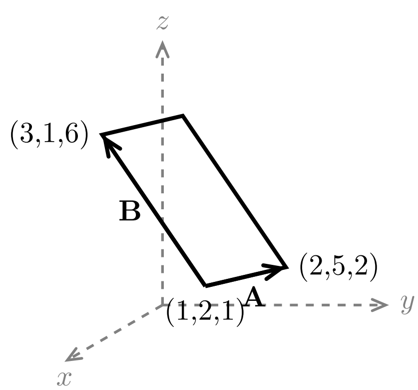
b) Using determinants we get

$$(\mathbf{i}+2\mathbf{j}) \times (2\mathbf{i}-3\mathbf{j}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 2 & -3 & 0 \end{vmatrix} = \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(-7) = -7\mathbf{k}.$$

Multiplying directly and using $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ etc we get

$$(\mathbf{i}+2\mathbf{j}) \times (2\mathbf{i}-3\mathbf{j}) = \mathbf{i} \times \mathbf{i} - 3\mathbf{i} \times \mathbf{j} + 4\mathbf{j} \times \mathbf{i} - 6\mathbf{j} \times \mathbf{j} = \mathbf{0} - 3\mathbf{k} - 4\mathbf{k} - 6 \cdot \mathbf{0} = -7\mathbf{k}.$$

2. Find the area of the parallelogram shown.



<u>**Answer:**</u> The area is $|\mathbf{A} \times \mathbf{B}|$, where $\mathbf{A}$ and $\mathbf{B}$ are the vectors along two adjacent edges of the parallelogram. These vectors are

$$\mathbf{A} = \langle 2,5,2\rangle - \langle 1,2,1\rangle = \langle 1,3,1\rangle \quad \text{and } \mathbf{B} = \langle 3,1,6\rangle - \langle 1,2,1\rangle = \langle 2,-1,5\rangle.$$

We computed this cross product in problem (1a). So,

$$\text{area } = |\langle 16,-3,-7\rangle| = \sqrt{256+9+49} = \sqrt{314}.$$