Geometry of systems of equations

1. Write a 3-by-3 system of equations

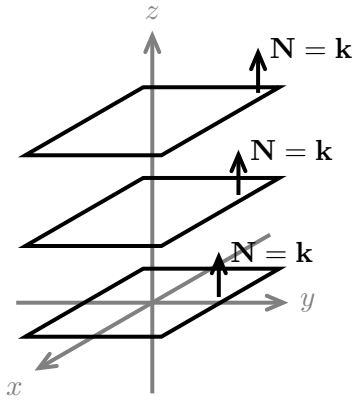
a) with no solutions and where all the planes are parallel;

b) where two planes are parallel and the other intersects them;

c) where the planes are all different and all intersect in a line.

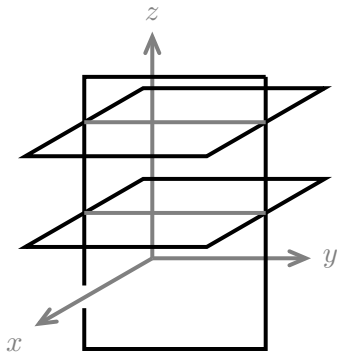
Answer: a) Planes are parallel if there normals are parallel. Here are two examples of such a system. We show a sketch of the second one.

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & 5 \\ x & + & 2y & + & 3z & = & 7 \\ x & + & 2y & + & 3z & = & 9 \end{array} \qquad \text{and} \qquad \begin{array}{l} z = 0 \\ z = 2 \\ z = 4 \end{array}$$



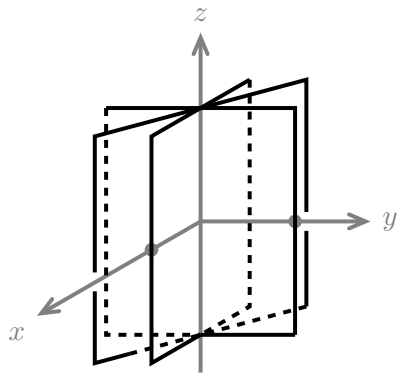
b) If planes are not parallel then they intersect, so it is easy to find many examples of this. Here are two, with a sketch of the second one.

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & 5 \\ x & + & 2y & + & 3z & = & 7 \\ x & + & y & + & z & = & 0 \end{array} \qquad \text{and} \qquad \begin{array}{l} z = 1 \\ z = 3 \\ x = 0 \end{array}$$



c) This is a little trickier. We'll use a lot of zeros to help. The following system intersects in the z-axis

$$\begin{array}{c} x = 0 \\ y = 0 \\ x + y = 0 \end{array}$$



18.02SC Multivariable Calculus
Fall 2010