Partial derivatives

1. Let $f(x,y) = \mathrm{e}^{(x^2+y^2)} + x^2 + y^2 + xy + 2y + 3$.

a) Compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

b) Show the second partials can be computed in any order. That is,

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

c) Find $\dfrac{\partial f}{\partial x}(1,3)$.

Answer: a) $\dfrac{\partial f}{\partial x} = 2x\mathrm{e}^{(x^2+y^2)} + 2x + y, \quad \dfrac{\partial f}{\partial y} = 2y\mathrm{e}^{(x^2+y^2)} + 2y + x + 2.$

b) To compute $\dfrac{\partial^2 f}{\partial x \partial y}$ we compute the partial with respect to x of $\dfrac{\partial f}{\partial y}$.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(2y\mathrm{e}^{(x^2+y^2)} + 2y + x + 2\right) = 4xy\mathrm{e}^{(x^2+y^2)} + 1.$$

Likewise

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(2x\mathrm{e}^{(x^2+y^2)} + 2x + y\right) = 4xy\mathrm{e}^{(x^2+y^2)} + 1.$$

We have shown the order of differentiation didn't matter.

c) Evaluating $\dfrac{\partial f}{\partial x}(1,3) = 2\mathrm{e}^{10} + 5.$