Tangent approximation

1. a) Find the equation tangent plane to the graph of $z = x^2 + y^2$ at the point (2,1,5).

b) Give the tangent approximation for z near the point $(x_0, y_0) = (2, 1)$.

Answer: a) $\dfrac{\partial z}{\partial x} = 2x$ and $\dfrac{\partial z}{\partial y} = 2y \Rightarrow \dfrac{\partial z}{\partial x}(2, 1) = 4$ and $\dfrac{\partial z}{\partial y}(2, 1) = 2$.

The tangent plane at (2,1,5) is

$$(z - 5) = \left.\frac{\partial z}{\partial x}\right|_0 (x - 2) + \left.\frac{\partial z}{\partial y}\right|_0 (y - 1) = 4(x - 2) + 2(y - 1).$$

b) The tangent approximation is the same formula, with the interpretation that for a fixed (x_0, y_0) the value of z on the graph of the function is near that of z on the tangent plane. Thus, for $(x_0, y_0) \approx (2, 1)$ we have

$$\Delta z \approx 4\Delta x + 2\Delta y.$$

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