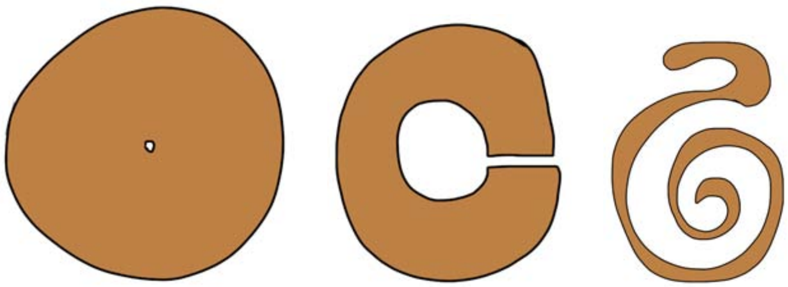
Problems: Simply Connected Regions

1. Which of the regions shown below are simply connected?



(a) (b) (c)

Answer: Region (a) is not simply connected – the "puncture" at the center of the disk would prevent any simple closed curve around it from contracting to a point while remaining within the region.

Regions (b) and (c) are simply connected. We frequently "cut" a region like (a) to create a simply connected region similar to (b). Region (c) illustrates the fact that simply connected regions aren't always simple!

For each of the vector fields described below, find the domain on which it is defined and continuously differentiable. Is that domain simply connected?

2. $\sin(x^2 + y^2)\mathbf{i} + \cos(x^2 + y^2)\mathbf{j}$

Answer: The vector field is defined and differentiable at all points (x, y). This region is simply connected.

3. $|x|\mathbf{i} + 0\mathbf{j}$

Answer: This vector field is not differentiable when $x = 0$. The region is the union of the open left and right half-planes and is not simply connected.

4. $\dfrac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$

Answer: The region is the punctured plane $(x, y) \neq (0, 0)$. It is not simply connected.

5. $\dfrac{y\mathbf{i} - x\mathbf{j}}{y^2}$

Answer: The region is the union of the upper and lower half planes; $y \neq 0$. It is not simply connected.

6. $\sqrt{x^2 - 1}\,\mathbf{i} + \sqrt{y^2 - 1}\mathbf{j}$

Answer: This vector field is defined and continuously differentiable when x^2 and y^2 are greater than 1. This region is the plane minus a square of side length two and is not simply connected.

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