Here again are 5 of my all-time favorites from this early in the term. Their past solutions will probably be floating around this Institute somewhere, but do spare yourself the trouble of searching those out, since (a) I will soon have fresh copies for you anyway, (b) noted educators maintain that you will probably learn a lot more by not cribbing so blatantly — or at least not until you too have become old and senile — and (c) those past answers may no longer be valid in this New Millenium.   AT

① Use complex arithmetic (rather than your calculator) to demonstrate that

$$z = 2 \cos 36° = e^{i\pi/5} + e^{-i\pi/5}$$

is one solution of $z^4 - 3z^2 + 1 = 0$.

② Perhaps from a sketch of two to illustrate the complex sums involved, determine the **net** increase (by 2π, 7.5π, or what?) in the argument or "polar angle" of

$$P(z) = z^7 + 5z^2 + 2 = (z-z_1)(z-z_2)\ldots(z-z_7)$$

as one travels counterclockwise once around the circle $|z| = 1$. Therefore **how many** of the seven roots $z_1, z_2, \ldots, z_7$ of this polynomial probably lie **inside** that unit circle? Do explain!

③ Show that the function $w(z) = z + 1/z$ maps any circle $|z| = $ const from the z-plane into an ellipse in the w-plane , with foci at $w = +2$ and $w = -2$.

④ For any real x evaluate the sum $S(x) = \sum_{k=-5}^{+5} e^{ikx}$ as the ratio of two sines.

⑤ Preferably via a **complex Newton** iteration programmed (by yourself!) into your calculator or computer, locate to at least 6 decimals that root of

$$e^z = z - 1$$

which lies closest to the origin in the first quadrant.