Probability: Terminology and Examples

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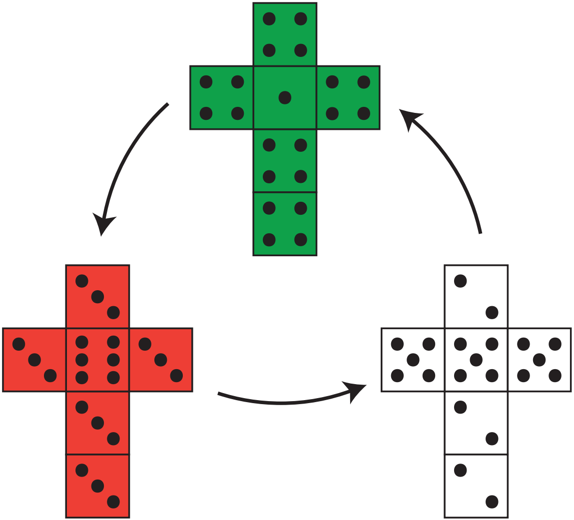
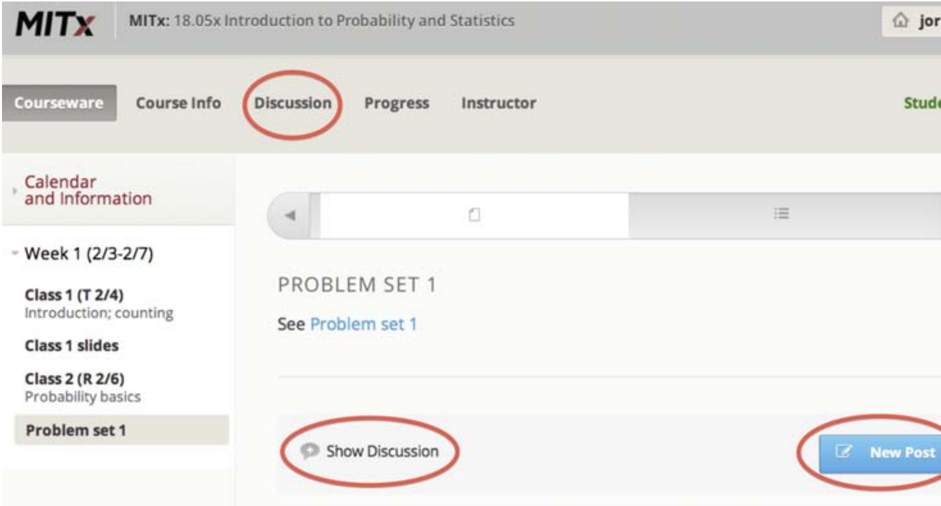


Discussion Boards



Board Question

Deck of 52 cards

- 13 *ranks*: 2, 3, . . . , 9, 10, J, Q, K, A
- 4 *suits*: $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$,

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other rank
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, K\diamondsuit\}$

Question

a) How many different 5 card hands have exactly one pair?

Hint: practice with how many 2 card hands have exactly one pair.
Hint for hint: use the rule of product.

b) What is the probability of getting a one pair poker hand?

Clicker Test

Set your clicker channel to 41.

Do you have your clicker with you?

$$\text{No} = 0$$
$$\text{Yes} = 1$$

Probability Cast

Introduced so far

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes S (or Ω).
- Event: a subset of the sample space.
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in S$
    1. Probability is between 0 and 1
    2. Total probability of all possible outcomes is 1.

Example (from the reading)

Experiment: toss a fair coin, report heads or tails.

Sample space: $\Omega = \{H, T\}$.

Probability function: $P(H) = .5, \quad P(T) = .5.$

Use tables:

Outcomes	H	T
Probability	1/2	1/2

(Tables can really help in complicated examples)

Discrete sample space

Discrete = listable

Examples:

{a, b, c, d} (finite)

{0, 1, 2, ...} (infinite)

Events

Events are sets:

- Can describe in words
- Can describe in notation
- Can describe with Venn diagrams

Experiment: toss a coin 3 times.

Event:

You get 2 or more heads = { HHH, HHT, HTH, THH}

CQ: Events, sets and words

Experiment: toss a coin 3 times.

Which of following equals the event "exactly two heads"?

$A = \{THH, HTH, HHT, HHH\}$
$B = \{THH, HTH, HHT\}$
$C = \{HTH, THH\}$

(1) A (2) B (3) C (4) A or B

CQ: Events, sets and words

Experiment: toss a coin 3 times.

Which of the following describes the event $\{THH, HTH, HHT\}$?

(1) "exactly one head"
(2) "exactly one tail"
(3) "at most one tail"
(4) none of the above

CQ: Events, sets and words

Experiment: toss a coin 3 times.

The events "exactly 2 heads" and "exactly 2 tails" are disjoint.

(1) True (2) False

CQ: Events, sets and words

Experiment: toss a coin 3 times.

The event "at least 2 heads" implies the event "exactly two heads".

(1) True (2) False

Probability rules in mathematical notation

Sample space: $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$

Outcome: $\omega \in S$

Probability between 0 and 1:

Total probability is 1:

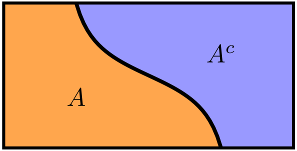
Event A: $P(A)$

Probability and set operations on events

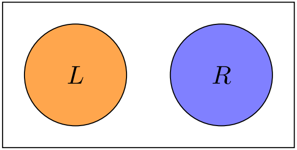
Rule 1. Complements. .

Rule 2. Disjoint events.

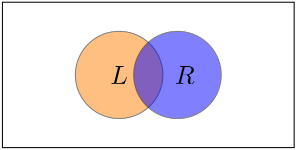
Rule 3. Inclusion-exclusion principle.



$\Omega = A \cup A^c$, no overlap



$L \cup R$, no overlap



$L \cup R$, overlap $= L \cap R$

Concept question

- Class has 50 students
- 20 male (M), 25 brown-eyed (B)

For a randomly chosen student what is the range of possible values for $p = P(M \cup B)$?

(a) $p \leq .4$
(b) $.4 \leq p \leq .5$
(c) $.4 \leq p \leq .9$
(d) $.5 \leq p \leq .9$
(e) $.5 \leq p$

Table Question

Experiment:

1) Roll your 20-sided die.

2) Check if all rolls at your table are distinct.

Repeat the experiment five times and record the results.

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1) Roll your 20-sided die.

2) Check if all rolls at your table are distinct.

Repeat the experiment five times and record the results.

For this experiment, how would you define the sample space, probability function, and event?

Compute the exact probability that all rolls are distinct.

Concept Question

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same
2. Different
3. It doesn't matter, same and different are equally likely

Board Question

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

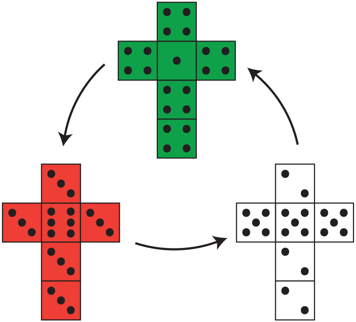
1. Same   2. Different   3. Doesn't matter

Question: Let p be the probability of heads and use probability to answer the question.

(If you don't see the symbolic algebra try p = .2, p=.5)

Jon's dice

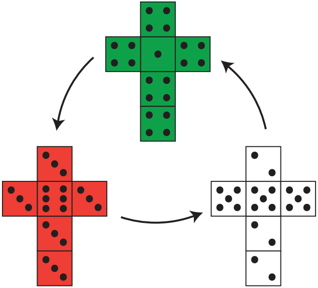
Jon has three six-sided dice with unusual numbering.



A game consists of two players each choosing a die. They roll once and the highest number wins.

Which die would you choose?

Board Question



Compute the exact probability of red beating white.

1) For red and white dice make the probability table.
2) Make a prob. table for the product sample space of red and white.
3) What is the probability that red beats white?

Answer to board question

For each die we have a probability table

	Red die	
Outcomes	3	6
Probability	5/6	1/6

	White die	
Outcomes	2	5
Probability	1/2	1/2

For both together we get a 2×2 probability table

		White	
		2	5
Red	3	5/12	5/12
	6	1/12	1/12

The red table entries are those where red beats white. Totalling the probability we get $P(\text{red beats white}) = 7/12$.

Concept Question

We saw red is better than white.

We'll tell you that white is better than green.

So red is better than white is better than green.

Is red better than green?