

Studio 8: NHST: t-tests and Rejection Regions
18.05 Spring 2014
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You should have downloaded studio8.zip and unzipped it into your 18.05 working directory.

Left-side vs Right-side

- Two groups: left-side of the class and right-side of the class.
- We will survey the heights (in inches above 5 feet) of everyone in each group.
- We want to test if the two groups have the same mean.
- While we compile the data you should answer the following questions.

Problem 1 (a) Use your favorite search engine to find the formula for running a two-sample t-test. Get the formulas for tests with equal and unequal variances.

(b) Find the function in R that will do this for you.

(c) Carefully list the assumptions needed for the t-tests in part (a).

Two-sided t-tests answers

answer: 1(a) Let n_L , m_L and s_L^2 be the number of people, sample mean and sample variance of the left-side group. Likewise n_R , m_R , s_R^2 .

In both cases we have the test statistic is $t = \frac{m_L - m_R}{s}$

(i) Variances of the two groups are equal:

$$s = \sqrt{\frac{(n_L - 1)s_L^2 + (n_R - 1)s_R^2}{n_L + n_R - 2}} \sqrt{\frac{1}{n_L} + \frac{1}{n_R}} \quad \text{and} \quad df = n_L + n_R - 2$$

(ii) Variance may not be equal (Welch's two-sample t-test):

$$s = \sqrt{\frac{s_L^2}{n_L} + \frac{s_R^2}{n_R}} \quad \text{and} \quad df = \frac{(s_L^2/n_L + s_R^2/n_R)^2}{(s_L^2/n_L)^2/(n_L - 1) + (s_R^2/n_R)^2/(n_R - 1)}$$

Two-sample t-test continued

answer: (1b) The R function `t.test` will do both types of two-sample t-tests. It has a boolean parameter `var.equal`.

answer: 1(c) The assumption is that both sets of data are drawn from normal distributions. The 'equal variance' test assumes the variances are the same in both distributions.

- In the equal variance test the t statistic exactly follows a t -distribution with the specified degrees of freedom.
- In the unequal variance test the t statistic is known to approximately follow a t -distribution with the specified degrees of freedom. Note that the degrees of freedom may not be an integer.

Run the tests in R

Problem 2. Use the data we just collected to run the tests:

(a) Code the tests by hand.

(b) Use the R function `t.test`. Compare these answers with part (a).

Both cases:
$$t = \frac{m_L - m_R}{s}$$

(i) Equal variances:

$$s = \sqrt{\frac{(n_L - 1)s_L^2 + (n_R - 1)s_R^2}{n_L + n_R - 2}} \sqrt{\frac{1}{n_L} + \frac{1}{n_R}} \quad \text{and} \quad df = n_L + n_R - 2$$

(ii) Unequal variances:

$$s = \sqrt{\frac{s_L^2}{n_L} + \frac{s_R^2}{n_R}} \quad \text{and} \quad df = \frac{(s_L^2/n_L + s_R^2/n_R)^2}{(s_L^2/n_L)^2/(n_L - 1) + (s_R^2/n_R)^2/(n_R - 1)}$$

Binomial rejection regions

Suppose we run an experiment counting the number of successes in 10 trials, so the outcomes follow a binomial(10, θ) distribution.

Let $H_0 = \theta = 0.4$.

Here is the table of binomial(10, 0.4) probabilities, i.e. the null distribution.

x	0	1	2	3	4	5	6	7	8	9	10
p	.006	.040	.121	.215	.251	.201	.111	.042	.011	.002	.000

Choose a two-sided rejection region for H_0 with significance level $\alpha = 0.1$.

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(We chose the left and right sides of the rejection region to be each as close to $\alpha/2 = 0.05$ as possible.)

Off-by-one errors

Example 0. How many examples are on this slide?

answer: 4

Example 1. If I make a reading assignment of pages 223 to 237 then how many pages does my class have to read?

answer: 14 pages. Notice that 223 to 223 is one page and 223 to 224 is two pages, so 223 to 237 is 14 pages.

Example 2. How many values does a binomial(12,.5) distribution take.

answer: 13. The values are 0, 1, 2, ..., 12 which contains 13 numbers.

Example 3. In R: `qbinom(0.05, 10, 0.04) = 2`. Suppose $X \sim \text{binomial}(10, 0.04)$. Is $P(X \leq 2)$ more or less than 0.05?

answer: More than 0.05. `qbinom` gives the smallest possible value with *at least* probability 0.05 to its left.

Binomial rejection regions using R

Problem 3. Data is drawn from a binomial(50, θ) distribution. Use `qbinom()` to find a two-sided rejection region for the null hypothesis that $\theta = 0.5$

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answer: (See `studio8-sol.r` for code.)

left-side = $[0,18]$, right-side = $[32,50]$

Lefties

The following reference says that about 11% of people are left-handed:

http://en.wikipedia.org/wiki/Handedness#cite_note-Papadatou-3

Is the handedness of MIT students the same as that for the global population?

- If you are left-handed please raise your hand.

Problem 4. (a) Use the data we just collected to test the null hypothesis that handedness among MIT's students is the same as that for the global population. Test at the 0.05 significance level.

(b) What assumptions did you make?

(c) It is believed that left-handed people are more likely to become scientists and engineers, so a one-sided alternative hypothesis could have been applied. Repeat the test as a one-sided test.

Solution to problem 4

answer: (a) See studio8-sol.r for code. Note, the code shows both an exact binomial test and a z-test.

(b) We assumed that our sample of students in this class is representative of MIT as a whole. For the z-test we assumed the sample size was large enough that, by the CLT, $\text{binomial}(n, .11)$ is approximately normal.

(c) See studio8-sol.r

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18.05 Introduction to Probability and Statistics

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