PROFESSOR: Hi, guys. My name is Nikola, and in this video, we're going to work out an example of an orthogonal projection matrix. Specifically, we are gonna compute the projection matrix onto the plane given by the equation x plus y minus z equals 0. So before we start, let me just recall what a projection matrix is.

So you've seen this sketch here a million times already. A projection matrix takes any vector in three-space-- well, just in this case we are dealing with a three-space-- and projects it down onto the plane, a two dimensional subspace of our three. So I'll give you a few moments to consider the problem for yourselves. And then you'll see my take on it.

Hi again. So in lecture, Professor Strang derived, in meticulous detail, the formula for the projection matrix. So it was given by the following slightly complicated expression. It's A times A transpose A inverse A transpose where A is a matrix that somehow encodes the subspace we're projecting on.

In particular, A has, as its columns, A1, A2-- I'm going to denote them-- a basis for the plane we're projecting on. So essentially, what we need to do is find two such vectors that span the plane and start computing with a matrix. This is fairly straightforward.

One choice that works, for example, is 1, -1, 0 for the first column, and 1, 0, 1 for the second column. And let me write out the matrix A. So in the formula, the slightly more complicated combination, this A transpose A inverse, so let me compute that first for you.

So A transpose A is a 2 by 2 matrix. And it's not so hard to figure out that it's 2 and 1, 1, 2. Now we shall invert it using the familiar formula 1 over the determinant. 2 times 2 minus 1 is 3.

And so we switch the diagonal entries, and we flip the signs of the other diagonal ones. Right. And therefore, projection matrix given by the following product of matrices 1/3, 2, -1, -1, 2, and then transpose of A, which is 1, -1, 0, 1, 0, 1.

I'm gonna carry out this multiplication in inhumanly fast fashion. So I'm just gonna write down the answer, which is 1/3, 2, -1, 1, -1, 2, 2, 1, 1, 1. So what you can do now is-- well, you can check whether this answer actually makes sense.

One thing you can do is just-- well, a projection matrix is supposed to take the normal to the plane down to 0. So you can just multiply P and the normal vector 1, 1, -1. And if you get 0, maybe you've done a good job.

Another curious thing that I would like to point out here is so you see we had lots of freedom choosing the matrix A. We could have chosen any two columns that span the subspace, that spans the plane. The beautiful thing about it is that in the end, we'll get the same answer.

So I hope there will be many of you who would say, hey, there is an easier way to do the problem. And I'll agree with these people. So let's see what would be an easier approach.

Well, let's go back to the sketch here. And let's make the following observation, that any vector is a sum of two components. The first component is its projection onto the plane. And the other component is its projection onto the orthogonal complement of the plane, in this case, onto the normal vector through the plane.

So in the language of linear algebra, this is just b equals to its projection onto the plane plus its projection-- I'm gonna call it PN-- onto the orthogonal complement of the plane. I'm gonna suggestively write here the identity matrix so that you can immediately read off a matrix equality. Associated with this equality here, it's the identity equals P plus PN. And therefore, the projection matrix is just the identity minus the projection matrix onto the normal vector.

Now, this object here, PN, is much easier to compute, well, for two reasons. First one is that projecting onto a one dimensional subspace is infinitely easier than projecting onto a higher dimensional subspace. And second, we already have-- well, immediately we can read off from the equation of the plane where the normal vector is. So we don't have derive these guys. We don't have to do what we did here.

So essentially, PN will be N N transpose N inverse N transpose. And that's equal to 1, 1, -1. And N transpose N inverse, this is just a number. It's 1 over the magnitude of the normal vector, so that's-- then, the magnitude squared. So that's 3 and 1, -1.

I'm gonna-- The answer here, it's 1/3, 1, 1, -1, 1, 1, -1, and -1, 1, 1. And in order to get the projection matrix-- yes?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Oh. Thank you. Thank you. And in order to get the projection matrix, we just subtract from the identity this expression. And you can confirm that it's-- we get the same answer as here. I think we're done here.