Pak

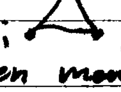
9/12/05

HW1 posted online (late last night), due 9/21 (W, since "student holiday" on M)

Thm (van der Woerden)
$\forall k, \ell$, $\exists\, n = n(k,\ell)$ s.t. every k-coloring of $[n]$ contains a length ℓ monochromatic arithmetic progression

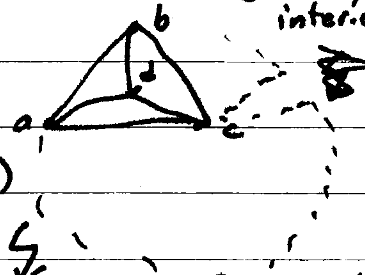
"we didn't go into bounds, but if you look, it just becomes towers of 2s all over the place"

Thm $n(2,\ell) \geq 2^{\ell/2}$ (pf from Jukna p. 230)
Pf: Color $[n]$ randomly. Pr(given ℓ-term seq. is mono) $= 2^{1-\ell}$
\# such progressions $< \binom{n}{2}$ (1st + second in seq.)
So prob(∃ mono) $< 2^{1-\ell}\binom{n}{2}$, so ETST $2^{1-\ell}\binom{n}{2} < 1$,
$n = 2^{\ell/2} \Rightarrow$ ✓

Thm (Erdős - Szekeres, 1935) $\forall k\, \exists\, n = n(k)$
$\forall$ n-point sets in $\mathbb{R}^2$ in general position, $\exists$ a k-subset in convex position.
general position: no 3 in same line
$n(3) = 4$ $n(4) = 9$ (?) ...
Pf: $n(k) = R_2(3,k)$ (edges = 3-subsets, 2 colors, want K_k)
First proof: color triangles by orientation ,
$i<j<r \Rightarrow$ one color, $i<r<j \Rightarrow$ ~~another~~ other. ~~If~~ Then mono subset is convex, since o/w  one of inner triangles contradicts clockwise- or counterclockwise-ness

Second pf (Jukna p327): Let $\alpha(a,b,c)$ = # of interior pts of ${}_a\triangle^b_c$

$$\delta(a,b,c) = \begin{cases} 1 & \alpha(a,b,c) \equiv_2 0 \\ 0 & o/w \end{cases}$$



$$\alpha(abc) = \alpha(abd) + \alpha(adc) + \alpha(acd) + 1$$

(in fact, this also tells you all subtriangles have same parity)

Def'n $\mathcal{F} = \{A_1, A_2, \dots\}$ $A_i \subset [n]$

$\mathcal{F}$ is K-colorable if $\exists\, \chi : [n] \to [K]$ s.t. $\forall i$ A_i is not monochromatic

Thm If $|A_i \cap A_j| \neq 1$ $\forall A_i, A_j \in \mathcal{F}$ then $\mathcal{F}$ is 2-colorable

Def'n $\mathcal{F}$ is K-uniform if $\forall i$ $|A_i| = K$

Pf: (greedy) (✓) (b/c o/w $\exists\, i$ s.t. $i \in A_p$, $i \in A_q$, A_p all red but i, A_q all blue but i)