PaK
9/21/05

in general, can hand in HW whenever before deadline,
if he's not in his office, just slide it under door.

Thm (5-color thm) (aaaw...)
Every planar graph can be colored w/ 5 colors

Pf: Let me make a

Lemma: G planar on n vertices $\Rightarrow$ G has at most $3n-6$ edges
Proof: terribly obvious

Corollary: (Useful to new HW!) (Thank goodness!)
G planar $\Rightarrow \exists$ vx w/ deg ≤ 5
Pf: Suppose not. ✓

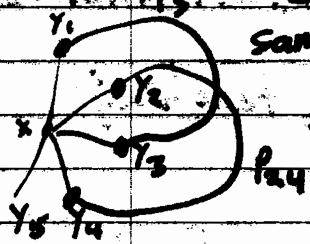
Rather unexpected Corollary ("6-color thm")
Pf: By contradiction. Suppose G is smallest counterexample. Then $\exists x \in V(G)$ s.t. deg $x \leq 5$.
$G - x = H$, color H w/ 6 colors, then color x ✓

Let me write Proof of 5-color thm

G min'l counter ex, $x \in V(G)$ deg 5
(if deg < 5, can disprove as above)
$H = G - x$, $\chi : H \to [5]$ proper coloring
$\Rightarrow |\chi(N(x))| = 5$

$H_{13} \subset H \qquad H_{13} = \chi^{-1}(\{1,3\}) \quad y_1 \to y_3$

P_{13} If colored 13 neighbors of x aren't in same component of H_{13}, ✓.



so $\exists$ path P_{13}, P_{24} ↯ ✓

Vizing's Thm

G a graph, $E = E(G)$

Def'n: An edge coloring is a map $\chi : E \to [N]$ s.t. adjacent edges have diff't colors

~~Thm~~

Def'n $\eta(G) = \min N$ s.t. $\chi : E \to [N]$ exists

Thm $\eta(G) \le K+1$ (also trivially $K \le \eta(G)$) where K = max deg of G.

Pf: color recursively adding one edge at a time. Note that K = max deg $\Rightarrow \exists$ at least one missing color at each vertex. Say you're now adding $e = (x, y_1)$. Look at y_1's missing color t_1, find edge from x w/ color t_1, $e_1 = (x\, y_2)$, look at y_2's missing color t_2, find e_2, etc. Keep going until you stop. Why you might stop: no edge colored t_r. Then color xy_1 t_1, xy_2 t_2, ... xy_r t_r ✓

The remaining so many minutes we devote to case 2

Case II: For some $i < r$ ~~missing~~ xy_i is t_r. In this case, do same recoloring as before up to xy_i. Now consider $H = G - X$

$H(s, t_i)$ = graph of edges w/ colors s (missing color) and $t_i = t_r$. $H(s, t_i)$ has deg ≤ 2. Also deg $x, y_i, y_r = 1$.

To be continued... (also in MGθ p15 [illegible])