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# Pattern Avoidance

$\omega \in S_k$

Def'n $\sigma \in S_n$ contains $\omega$ if $\exists\, i_1 \dots i_k$ s.t.
$\sigma(i_p) < \sigma(i_q)$ iff $\omega(p) < \omega(q)$

$\sigma$ avoids $\omega$ otherwise

Thm
$\forall \omega \in S_3$ # $\omega$-avoiding permutations $\sigma \in S_n$ is
$C_n = \frac{1}{n+1}\binom{2n}{n}$

Pf: Note that, by subtracting from $n+1$ or reversing order, {231-avoiding} $\cong$ {312 avoiding} etc.
and {321-avoiding} = {123 avoiding}

So just consider ~~231~~ 312 + 123.

312 ~~231~~: $\sigma$ ~~231~~ 312 avoiding $\Rightarrow$ $\sigma^{-1}(1)$ is in the middle somewhere, everything to the ~~right~~ left must be smaller than everything to the right (o/w have 312 $\subseteq$). Build binary tree w/ root 1, left child is min on left, right child is min on right, etc.

# binary trees = $C_n$ ✓

321 avoiding: Dilworth $\Rightarrow$ $\sigma$ can be decomposed into two increasing subsequences. Then RSK-corresp. to build pair of tableaux, convert those into lattice paths to get a bijection between 321-avoiding + Dyck paths

So $\omega$-avoiding is $e^{O(n)}$ "Stanley-Wilf conjecture"

(proof next time)