18.465 4th problem set due Tuesday, March 29, 2005

1. For a given n, let j be the greatest integer $\leq n/4$ and let k be the smallest integer $\geq 3n/4$. Then one possible definition of the *interquartile range* is $X_{(k)} - X_{(j)}$. Find the breakdown point of this statistic, and its limit as $n \to \infty$.
2. Randles and Wolfe p. 246 Problem 7.4.1 (show that the inequality holds for all θ).
3. R&W p. 248 Problem 7.4.7.
4. In the handout "Breakdown points of 1-dimensional location M-estimators", it is stated that if k is an integer $\geq n/2$, then given $X_1, ..., X_n$, there exist $Y_1, ..., Y_n$ with $Y_i = X_i$ for $n - k$ values of i while $Y_1, ..., Y_n$ are symmetrically distributed around an arbitrarily large median, which is then the M-estimator. Prove these statements in detail.
5. Consider the data set $0, 1, 2, 2, 4.5, 5.6, 8.9$.

(a) Find the breakdown point of $1/S$ at this data set, where S is S^* as defined in R&W, (7.4.16).

(b) Compare the breakdown point of the M-estimator (which uses $S = S^*$) to that of $1/S$.