18.465 PS7 due Thursday, April 28, 2005

The first three of the following problems, and problem 6 for extra credit, are from p. 166 of the handout, Chapter 4 of UCLT.

1. P. 166, problem 1.

2. P. 166, problem 2.

3. P. 166, problem 5.

4-5. Let $\mathcal{H}$ be the collection of all half-planes in the plane, as described in problem 6 on p. 166.
(a) Find $S(\mathcal{H})$.
(b) Give an upper bound for $m^{\mathcal{H}}(n)$ based on part (a).
(c) In fact, $m^{\mathcal{H}}(n) = 2\,{}_{n-1}C_{\leq 2} = n^2 - n + 2$ for $n \geq 1$. Verify this for $n = 3$ and 4.
(d) An inequality of L. Devroye implies that for $M > 0$ and any VC class $\mathcal{C}$ with suitable measurability properties (which $\mathcal{H}$ has),

$$\Pr\{\sup_{C \in \mathcal{C}} \sqrt{n}|(P_n - P)(H)| \geq M\} \ \leq \ 4m^{\mathcal{H}}(n^2) \exp\left(-2M^2 + Mn^{-1/2} + 4M^2 n^{-1}\right).$$

Note: one can still see the factor $\exp(-2M^2)$ familiar from one dimension, and it remains the dominant factor in a sense, but note that the left side of the inequality becomes 0, so the inequality is trivial, if $M > \sqrt{n}$.

For $n = 100$ and $\mathcal{C} = \mathcal{H}$, how large must M be to make Devroye's bound less than 0.05? Is the M you find less than $\sqrt{n}$, so that the inequality is non-trivial?

6. (For extra credit.) P. 166, problem 8.