

Part (2)

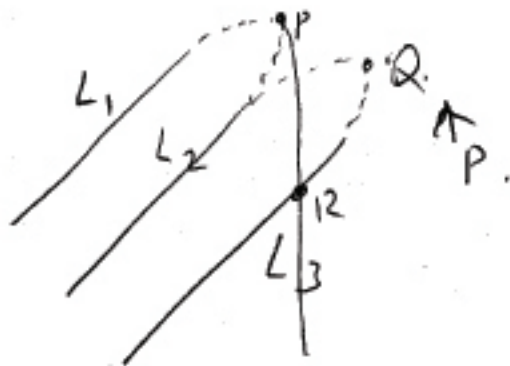
9/10/04

geometric

- ① Two distinct points lie on a unique line
- ② Two distinct lines intersect at a unique point.

Asking: How many lines pass through a point? How many lines intersect at a point?

Euclidean plane:  $\mathbb{A}^2$ .



Two lines intersect at a point

$$PR \parallel L_2$$

$$L_3 \parallel L_2$$

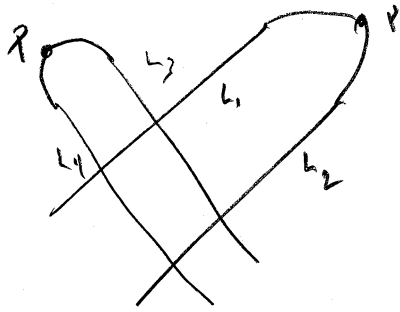
Two lines intersect at a point

"one point compactification" ...

Sketch

Suppose:

(write down basis of coordinates)



$$P, Q \in L_3$$

$$P, Q \in L_1$$

PROJECTIVE POINT: either a point in the plane OR a "direction" in the plane.

(directed graph)

$$\mathbb{P}^2 = \mathbb{A}^2 \cup \{\text{directions in } \mathbb{A}^2\}$$

live through origin.

book says this is  $\mathbb{P}^1$ .

A pair of opp. points on unit circle.

[Venn diagram]

$$Ay = Bx$$

$$[A, B]$$

that's where the  $\mathbb{P}^1$  come in.

(Projective)

# LINE

$\Rightarrow$  set of points in a planar line  $\cup$  {the direction of that line}

$$Ax + By + C = 0$$

(vertical asymptote of plane is necessary)

$$Ax + By = 0$$

(Direction = )  $[A, B]$   
differential

OR

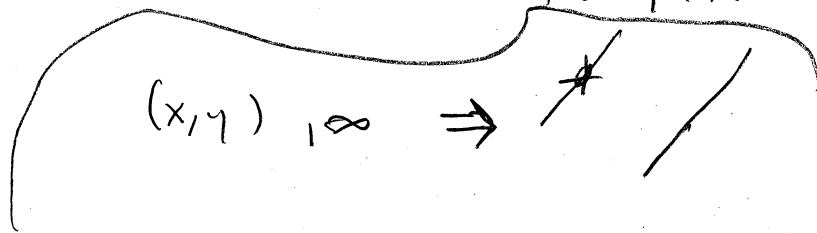
$$L_\infty = \{ \text{all points at } \infty \}$$

① Check:

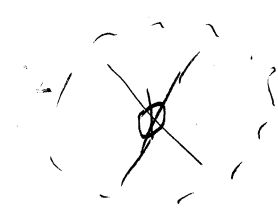
$$(x, y), (z, w) \in A^2$$

$\Rightarrow$  planar line through it.

This



$\infty_1, \infty_2$



... between ... that ...

$$[a, b, c]$$

$$\text{If } c \neq 0, \text{ then } \sim \left[ \frac{a}{c}, \frac{b}{c}, 1 \right] = [x, y, 1].$$

$$\text{If } c = 0 \text{ then } = [A, B, 0]$$

$$\iff (x, y) \in \mathbb{A}^2$$

$$\iff [A, B] \text{ (direction)}$$

$$\left\{ [x, y, z] \mid \alpha x + \beta y + \gamma z = 0 \right\}$$

↑      ↑      ↑  
coordinates.

at least one of  $\alpha, \beta \neq 0$ .

$$\rightarrow [a, b, c] \in L, c \neq 0 \text{ then this gives } \left[ \frac{a}{c}, \frac{b}{c}, 1 \right]$$

↓  
 $\left[ \frac{a}{c}, \frac{b}{c} \right]$

which is on the line  $\alpha x + \beta y + \gamma = 0$ .

$$\rightarrow [a, b, 0] \in L \Rightarrow = [-\beta, \alpha, 0]$$

↓  
 $-\beta y = \alpha x$

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