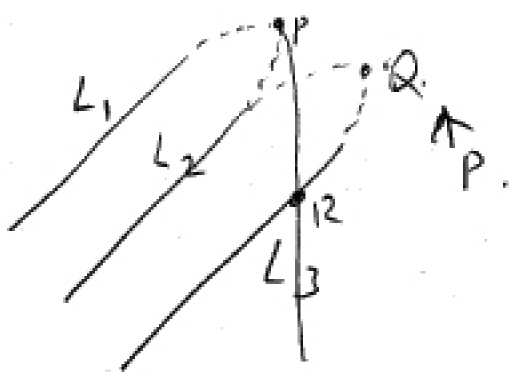
Part (2) 9/10/04

Geometric

1. Two distinct points lie on a unique line
2. Two distinct lines intersect at a unique point.

Euclidean plane: $\mathbb{A}^2$.

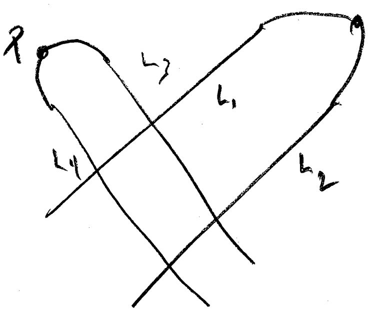


$PR \parallel L_2$

$L_3 \parallel L_2$

"one point compactification" ..

Suppose:



$$P, Q \in L_3$$
$$P, Q \in L_1$$

PROJECTIVE POINT: either a point in the plane OR a "direction" in the plane.

(directed graph)

$\Updownarrow$

line through origin.

$\Updownarrow$

A pair of opp. points on unit circle.

$$\mathbb{P}^2 = \mathbb{A}^2 \cup \{\text{directions in } \mathbb{A}^2\}$$

book says this is $= \mathbb{P}^1$.

$Ay = Bx$ $\qquad [A, B]$

that's where the $\mathbb{P}^1$ come in.

(projective)

LINE

⇒ set of points in a planar line $\cup$ {the direction of that line}

$$Ax + By + C = 0$$

$$Ax + By = 0$$

Direction $=$ $[A, B]$

OR

$L_\infty = \{$all points at $\infty\}$.

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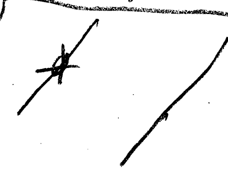
① Check:

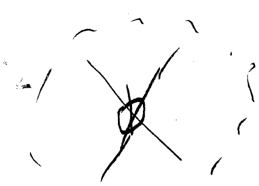
$(x,y), (z,w) \in A^2$

⇒ planar line through it.

This

$(x,y), \infty \Rightarrow$ 

∞_1, ∞_2



$[a, b, c]$

If $c \neq 0$, then $\sim \left[\frac{a}{c}, \frac{b}{c}, 1\right] = [x, y, 1]$.

If $c = 0$ then $= [A, B, 0]$

$\Longleftrightarrow \quad (x, y) \in \mathbb{A}^2$

$\Longleftrightarrow \quad [A, B]$ (direction)

$\{ [x, y, z] \mid \alpha X + \beta Y + \gamma Z = 0 \}$

coordinates.

at least one of $\alpha, \beta \neq 0$.

$\rightarrow [a, b, c] \in L$, $c \neq 0$ then this equals $\left[\frac{a}{c}, \frac{b}{c}, 1\right]$

$\downarrow$

$\left[\frac{a}{c}, \frac{b}{c}\right]$

Which is on the line $\alpha X + \beta Y + \gamma = 0$.

$\rightarrow [a, b, 0] \in L \Rightarrow = [-\beta, \alpha, 0] \mapsto -\beta Y = \alpha X$