18.950 Differential Geometry
 Fall 2008

18.950 Homework 8

1. (10 points) Let $M \subset \mathbb{R}^{n+1}$ be a compact hypersurface, and $p \in \mathbb{R}^{n+1} \setminus M$ a point. The winding number is defined to be the degree of the map

$$M \longrightarrow S^n, \quad y \mapsto \frac{y - p}{\|y - p\|}.$$

Prove the following: if the winding number is nonzero, every smooth path $c : [0, \infty) \rightarrow \mathbb{R}^{n+1}$ with $c(0) = p$, $\lim_{t \to \infty} \|c(t)\| = \infty$ must intersect M somewhere.

2. (10 points) Determine the degree of the map

$$\phi : S^2 \rightarrow S^2, \quad \phi(y_1, y_2, y_3) = \frac{(y_1^2 - y_2^2, 2y_1 y_2, y_3)}{\sqrt{(y_1^2 + y_2^2)^2 + y_3^2}},$$

(i) by computing the integral (computer-assistance is highly recommended), and (ii) using more advanced results from the lecture.