Case Study : Portfolio Theory

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1   Simulation: Two-Asset Portfolios

Consider $m = 2$ assets:

$$R_1 : E(R_1) = 0.15 = \alpha_1 \quad \sqrt{Var(R_1)} = 0.25 = \sigma_1$$
$$R_2 : E(R_1) = 0.20 = \alpha_2 \quad \sqrt{Var(R_1)} = 0.30 = \sigma_2$$

$$Corr(R_1, R_2) = \rho$$

Portfolio:

$$R_w = (1 - w)R_1 + wR_2, \quad 0 \le w \le 1$$

$$\alpha_w = E[R_w] = (1 - w)\alpha_1 + w\alpha_2$$

$$\begin{aligned}\sigma_w^2 &= Var(R_2) \\ &= (1 - w)^2\sigma_1^2 + w^2\sigma_2^2 + 2(1 - w)(w)\rho\sigma_1\sigma_2\end{aligned}$$

Mean-Variance Analysis

Feasible Portfolio Set:

$$\Pi^* = \{(\sigma_w, \alpha_w) : \quad 0 \le w \le 1\}$$

Issues:

- What is Π^*?

- What portfolios are optimal / sub-optimal?

- How to choose/specify an optimal portfolio?

- Do optimal portfolios have special structure?

Simulation:

- Simulate 500 weekly returns with
  $\rho = -.8, -.4, 0., +.4, +.8$

- Examine

  - Cumulative returns of each asset

– Asset returns: means, volatilities, correlations

– Plot of Π^*

– Cumulative returns of each asset and the minimum-variance portfolio.

See the plots in the pdf file $Simulation\_TwoAsset\_Portfolios.pdf$.

2 US Sector ETFs: 2009-2013

2.1 Mean, Variance, Correlation Statistics

```
Sector ETFs:  Period: 2009-2013


Annualized Return and Volatility:


                  Ret  Vol
MATERIALS(XLB)    0.16 0.24
HEALTH CARE(XLV)  0.17 0.15
CONSSTAPLES(XLP)  0.15 0.12
CONSDISC(XLY)     0.24 0.21
ENERGY(XLE)       0.15 0.25
FINANCIAL(XLF)    0.13 0.33
INDUSTRIALS(XLI)  0.18 0.23
TECHNOLOGY(XLK)   0.18 0.19
UTILITIES(XLU)    0.10 0.16



 Correlations:


                   XLB  XLV  XLP  XLY  XLE  XLF  XLI  XLK  XLU
MATERIALS(XLB)     1.00 0.70 0.68 0.87 0.88 0.74 0.90 0.85 0.59
HEALTH CARE(XLV)   0.70 1.00 0.80 0.72 0.69 0.64 0.76 0.69 0.64
CONSSTAPLES(XLP)   0.68 0.80 1.00 0.77 0.69 0.63 0.76 0.70 0.71
CONSDISC(XLY)      0.87 0.72 0.77 1.00 0.82 0.82 0.93 0.88 0.67
ENERGY(XLE)        0.88 0.69 0.69 0.82 1.00 0.74 0.85 0.80 0.65
FINANCIAL(XLF)     0.74 0.64 0.63 0.82 0.74 1.00 0.85 0.69 0.57
INDUSTRIALS(XLI)   0.90 0.76 0.76 0.93 0.85 0.85 1.00 0.85 0.66
TECHNOLOGY(XLK)    0.85 0.69 0.70 0.88 0.80 0.69 0.85 1.00 0.59
UTILITIES(XLU)     0.59 0.64 0.71 0.67 0.65 0.57 0.66 0.59 1.00
```

2.2 Optimal Portfolios (Max Allocation=0.30)

```
Optimal Allocations for Selected Target Vols
   Max. Allocation = 0.30
                  target.vol0.009 target.vol0.099 target.vol0.153
MATERIALS(XLB)              0.000           0.000           0.000
HEALTH CARE(XLV)            0.009           0.157           0.300
CONSSTAPLES(XLP)            0.040           0.300           0.227
CONSDISC(XLY)              0.017           0.226           0.300
ENERGY(XLE)                0.000           0.000           0.000
FINANCIAL(XLF)             0.000           0.000           0.000
INDUSTRIALS(XLI)           0.000           0.000           0.000
TECHNOLOGY(XLK)            0.000           0.000           0.173
UTILITIES(XLU)             0.000           0.000           0.000
riskFree                   0.935           0.317           0.000
          Portfolio Statistics for Optimal Allocations
             target.vol0.009 target.vol0.099 target.vol0.153
Ann Return             0.011           0.124           0.187
Ann Volatility         0.009           0.099           0.153
```

Graphical displays of the optimal allocations are presented in plots 2 and 3 of *ETFS_1_periodA_30.pdf*

- As the target return increases from zero, only XLY, XLP, XLV are in the model. They enter in the same proportion, i.e., the scaled (de-levered) optimal portfolio w/o constraints.

- When the allocation constraint is hit, first for consumer staples, higher allocations given to XLY and XLV

- When the .30 allocations are reached for these three, then XLK (tech) is added. It has higher return than the other ETFs, so eventually allocations to XLY and XLP are reduced to allow for higher-return from XLK.

- From the efficient frontier, all the ETFs (except XLY) are dominated by an optimal allocation with a 0.30 max constraint.

- No allocation is ever given to Financials (XLF).

2.3 Optimal Portfolios (Max Allocation=0.15)

```
Optimal Allocations for Selected Target Vols
        Max. Allocation = 0.15
                  target.vol0.009 target.vol0.099 target.vol0.163
MATERIALS(XLB)              0.000           0.000           0.098
HEALTH CARE(XLV)            0.009           0.150           0.150
CONSSTAPLES(XLP)            0.042           0.150           0.150
CONSDISC(XLY)               0.018           0.150           0.150
ENERGY(XLE)                 0.000           0.000           0.000
FINANCIAL(XLF)              0.000           0.000           0.000
INDUSTRIALS(XLI)            0.000           0.000           0.150
TECHNOLOGY(XLK)             0.000           0.150           0.150
UTILITIES(XLU)              0.000           0.069           0.150
riskFree                    0.931           0.331           0.002


Portfolio Statistics
               target.vol0.009 target.vol0.099 target.vol0.163
Ann Return           0.012           0.117           0.168
Ann Volatility       0.009           0.099           0.163
```

Graphical displays of the optimal allocations are presented in plots 2 and 3 of $ETFS\_1\_periodA\_15.pdf$

- The 0.15 maximum allocation constraint has no impact on low-return portfolios.

  The optimal portfolios allocate to XLP, XLY and XLV, initially until they hit their limits.

- The allocations to XLK increases until its limit is reached.

- The Allocation to XLU (utilities) is mixed with XLI (industrials), until their limits

- Efficient frontier with Max Allocation=0.30 is above the EF for Max Allocation =0.15

  Compare Plot 4 in the two files $ETFS\_1\_periodA\_30.pdf$ and $ETFS\_1\_periodA\_30.pdf$

3 US Sector ETFs: 2003-2006

3.1 Mean, Variance, Correlation Statistics

```
Sector ETFs:  Period: 2003-2006


Annualized Return and Volatility:
                  Ret  Vol
MATERIALS(XLB)    0.16 0.18
HEALTH CARE(XLV)  0.07 0.12
CONSSTAPLES(XLP)  0.08 0.09
CONSDISC(XLY)     0.14 0.14
ENERGY(XLE)       0.26 0.20
FINANCIAL(XLF)    0.15 0.13
INDUSTRIALS(XLI)  0.15 0.14
TECHNOLOGY(XLK)   0.12 0.18
UTILITIES(XLU)    0.19 0.13



 Correlations:
                  XLB  XLV  XLP  XLY  XLE  XLF  XLI  XLK  XLU
MATERIALS(XLB)    1.00 0.45 0.57 0.74 0.54 0.68 0.82 0.66 0.50
HEALTH CARE(XLV)  0.45 1.00 0.55 0.54 0.24 0.61 0.59 0.47 0.42
CONSSTAPLES(XLP)  0.57 0.55 1.00 0.69 0.16 0.71 0.65 0.50 0.45
CONSDISC(XLY)     0.74 0.54 0.69 1.00 0.31 0.82 0.85 0.79 0.48
ENERGY(XLE)       0.54 0.24 0.16 0.31 1.00 0.24 0.37 0.21 0.51
FINANCIAL(XLF)    0.68 0.61 0.71 0.82 0.24 1.00 0.78 0.70 0.54
INDUSTRIALS(XLI)  0.82 0.59 0.65 0.85 0.37 0.78 1.00 0.78 0.49
TECHNOLOGY(XLK)   0.66 0.47 0.50 0.79 0.21 0.70 0.78 1.00 0.40
UTILITIES(XLU)    0.50 0.42 0.45 0.48 0.51 0.54 0.49 0.40 1.00
```

3.2 Optimal Portfolios (Max Allocation=0.30)

```
Optimal Allocations for Selected Target Vols
   Max. Allocation = 0.30
```

	target.vol0.009	target.vol0.1	target.vol0.114
MATERIALS(XLB)	0.000	0.000	0.000
HEALTH CARE(XLV)	0.000	0.000	0.000
CONSSTAPLES(XLP)	0.004	0.088	0.093
CONSDISC(XLY)	0.000	0.000	0.000
ENERGY(XLE)	0.017	0.251	0.300
FINANCIAL(XLF)	0.013	0.213	0.270
INDUSTRIALS(XLI)	0.003	0.027	0.037
TECHNOLOGY(XLK)	0.000	0.000	0.000
UTILITIES(XLU)	0.042	0.300	0.300
riskFree	0.921	0.121	0.000

```
        Portfolio Statistics for Optimal Allocations
```

	target.vol0.009	target.vol0.1	target.vol0.114
Ann Return	0.015	0.165	0.188
Ann Volatility	0.009	0.100	0.114

Graphical displays of the optimal allocations are presented in plots 2 and 3 of $ETFS\_1\_periodB\_30.pdf$

- As the target return increases from zero, only XLU, XLE, XLF, XLP, and XLI are in the model. They enter in the same proportion, i.e., the scaled (delevered) optimal portfolio w/o constraints.

- The allocation constraint is hit first for utilities (XLU).

- As the target return increases, the energy (XLE) and financials (XLF) increase to their limits.

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- At high target return levels, allocation to industrials (XLI) substitutes for consumer stables (XLP).

3.3 Optimal Portfolios (Max Allocation=0.15)

```
Optimal Allocations for Selected Target Vols
   Max. Allocation = 0.15
                  target.vol0.01 target.vol0.1 target.vol0.11
MATERIALS(XLB)             0.000         0.022           0.092
HEALTH CARE(XLV)           0.000         0.000           0.000
CONSSTAPLES(XLP)           0.004         0.150           0.150
CONSDISC(XLY)              0.000         0.150           0.150
ENERGY(XLE)                0.019         0.150           0.150
FINANCIAL(XLF)             0.014         0.150           0.150
INDUSTRIALS(XLI)           0.003         0.150           0.150
TECHNOLOGY(XLK)            0.000         0.000           0.000
UTILITIES(XLU)             0.044         0.150           0.150
riskFree                   0.916         0.078           0.008
          Portfolio Statistics
                  target.vol0.01 target.vol0.1 target.vol0.11
Ann Return                 0.016         0.149            0.16
Ann Volatility             0.010         0.100            0.11
```

Graphical displays of the optimal allocations are presented in plots 2 and 3 of *ETFS_1_periodB_15.pdf*

- The 0.15 maximum allocation constraint has no impact on the relative allocations for low-return portfolios.

- Technology (XLK) never enters the optimal allocation.

- The allocations to XLK increases until its limit is reached.

- Efficient frontier with Max Allocation=0.30 is above the EF for Max Allocation =0.15. Compare Plot 4 in the two files $ETFS\_1\_periodB\_30.pdf$ and $ETFS\_1_periodB\_15.pdf$

  The curve of the latter starts to bend lower as the maximum allocation constraints are hit. As a result greater return is achieved, only by those assets that result in marginal increases in the relative volatility.

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