

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

13.013J/1.053J Dynamics and Vibration

Fall 2002

Problem Set 11

Issued: **Day 28**

Due: 11am, **Day 31**

Presentations:

1. (Problem 1)
(This was your problem last time—Show the whole solution from the last homework and for this one.)
2. (Problem 2)
3. (Problem 3)
4. (Problem 4)
5. (Problem 5)

- Think about these problems before class and come prepared to ask me about what you don't understand. However, read through the notes I handed out first.
- I am expecting well-prepared presentations. For the problem you are responsible for, I expect you to have really worked to get it right, including coming in to talk if you don't understand something.

1. You did problem 6-61 for the last problem set. This time, let $L=1.0$ m, $m = 2.0$ kg, and $f_n=1.3$ Hz.
 - a. Determine the spring constant, k .
 - b. In a free vibration test, two successive peaks, A_n and A_{n+1} , are found to have amplitudes of $A_n = 0.1$ radians, and $A_{n+1} = 0.03$ radians. Use the concept of the logarithmic decrement to calculate the damping ratio for the system.
 - c. Determine the dashpot constant, C .

2. For any single degree of freedom oscillator, the energy lost per cycle is related to the damping and to the logarithmic decrement. Show that in free vibration from initial conditions, the ratio of the energy lost per cycle to the initial energy is approximately given by $4\pi \cdot \text{damping ratio}$.

Hint: Assume a simple mass-spring dashpot system. Assume two successive peaks in a free vibration decay with amplitudes A_n and A_{n+1} . Compute the maximum potential energy for each peak. Take the ratio of the change in energy to the initial energy between the two peaks and relate this to the logarithmic decrement. You can read up on the log decrement in the notes I handed out or in the textbook.

3. We did the TLP in heave last time. The buoyancy spring constant was given by $K_b = \rho \cdot g \cdot A_{WL}$, where ρ is the density of water and A_{WL} is the area of the water line. The total tendon spring constant was given by $K_T = 6EA_{xc}/L_T$. The tendons are steel with a modulus of $E=30,000$ Kips. The OD and ID of a tendon are 28.0 and 26.1 inches. The operating draft is given as 91 feet and the total pretension on the risers is 9678 Kips. The stretched length of the tendon is 3204 feet.
 - a. Determine the unstretched length of the tendons.
 - b. How much ballast water would you have to take on board to lower the TLP in the water down to the point at which you could connect the unstretched riser?
 - c. Compute again the natural period of the TLP in heave at the tendon length of 3204 Feet. Then recompute the natural period for the same TLP except that the tendons are twice as long. I.e. $L_T = 6408$.
 - d. At which of these two periods are ocean waves likely to be larger and exert more excitation on the structure.

4. Assume the TLP is oscillating in heave at its natural frequency with an amplitude of A feet.
 - a. Find an expression for the peak kinetic energy of vibration in heave. Include only the kinetic energy associated with the hull and its added mass, but not the kinetic energy of the tendons.
 - b. Assume the tendons stretch linearly, that is zero at the bottom and maximum at the top end. Compute the maximum kinetic energy associated with the dynamic motion of the tendons and compare it to the maximum kinetic energy of the hull. Hint, do this by finding the velocity and kinetic energy of a small element of the tendon. Then get the energy of the whole tendon by integrating along the length.
 - c. Compute the ratio of the tendon kinetic energy to the hull kinetic energy.
 - d. When we computed the heave natural frequency of the TLP we neglected the mass of the tendons. Can you think of a way using the energies found above to find an approximate formula for the heave natural frequency, which accounts for the mass of the tendon?
5. Assuming small motions, the TLP acts like an inverted pendulum in surge and sway. Assume that the added mass in surge and sway is the same as the heave added mass, which was given as 60% of the total displaced mass of water. In surge and sway the hull moves horizontally but does not tilt. Hence you do not have to account for the kinetic energy of rotation. You need only approximate the pendulum as a point mass.
 - a. Obtain an estimate of the surge/sway natural period for the installed tendon length of 3204 feet.
 - b. By ignoring the mass of the tendons will we under or overestimate the natural frequency.