2.004 MODELING DYNAMICS AND CONTROL II Spring 2003

Problem Set No.1

Problem 1

An experimental apparatus to determine the friction characteristics of various materials is shown in Figure 1. A mass m is released with a known velocity and slides on the test sample. The velocity decay, due to the frictional resistance, is monitored and used to estimate the effective viscous drag coefficient. Derive the system equation of motion and determine the system time constant. Derive an expression for the velocity of the mass assuming that it is released with an initial velocity v_0 at time $t = 0$. If the mass is 10 kg, and after a time of 5.0 seconds the mass is at 50% of its initial velocity, determine the value of the damping parameter.

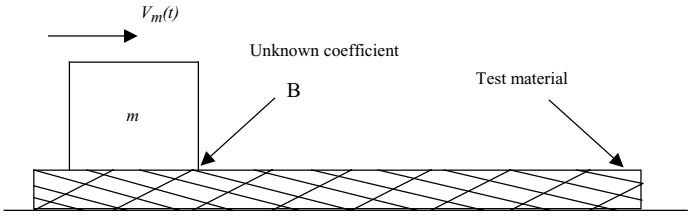

Figure 1: Friction coefficient test apparatus

Problem 2

In a manufacturing process a motor drives a surface grinding wheel with diameter d and mass m. The friction from the workpiece produces a torque load which is proportional to the wheel speed. The motor is driven by a current source, and produces a torque proportional to the input current $I_s(t)$. Derive the differential equation relating the grinding wheel speed to the input current. Determine the system time constant and determine the grinding wheel angular velocity as a function of time for the current input shown in Figure 2. Make a sketch of the angular velocity response.

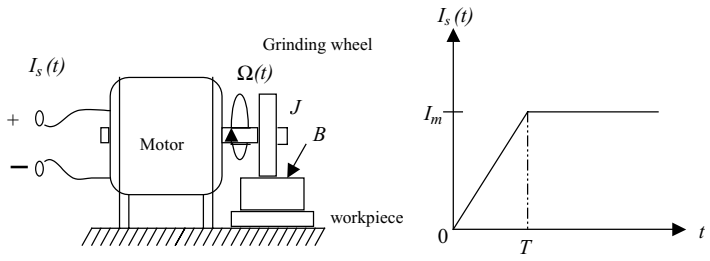

Figure 2: Surface grinder and excitation current waveform

Problem 3

A tracking radar is being designed to follow the motion of a satellite as shown in Figure 3. An automatic control system is desired to position the elevation angle ψ of the radar dish in proportion to an input or command voltage v. A *servomotor* is available which is represented by the following equation, in which the dish inertia is proportional to τ_s:

$$\tau_s(d^2\psi/dt^2) + (d\psi/dt) = K_v v_1,$$

where v_1 is the voltage applied to the motor.
It is proposed to measure ψ by an electrical device such that

$$v_\psi = K_\psi \psi,$$

and to let v_1 be proportional to the error $v - v_\psi$, where v is the input voltage:

$$v_1 = K(v - v_\psi),$$

thus feeding back the position signal to achieve a closed-loop control system.

(a)Find the differential equation relating ψ to input v.

(b)Find the system "steady-state gain" $= (\psi/v)_{t\to\infty}$.

(c)Find the undamped natural frequency and damping ratio of the complete system.

(d)Derive an expression for the value of K which will produce the fastest response to a step change in v without overshoot of the steady-state, $(\psi)_{t\to\infty}$. Estimate the time T for ψ to reach 90% of $(\psi)_{t\to\infty}$.

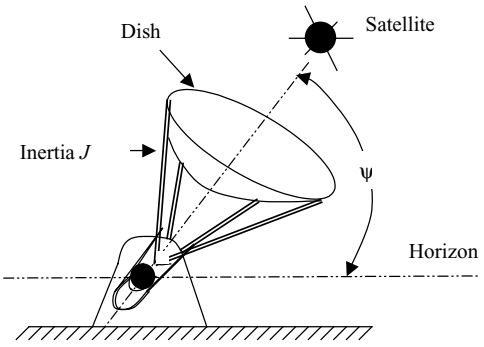

Figure 3