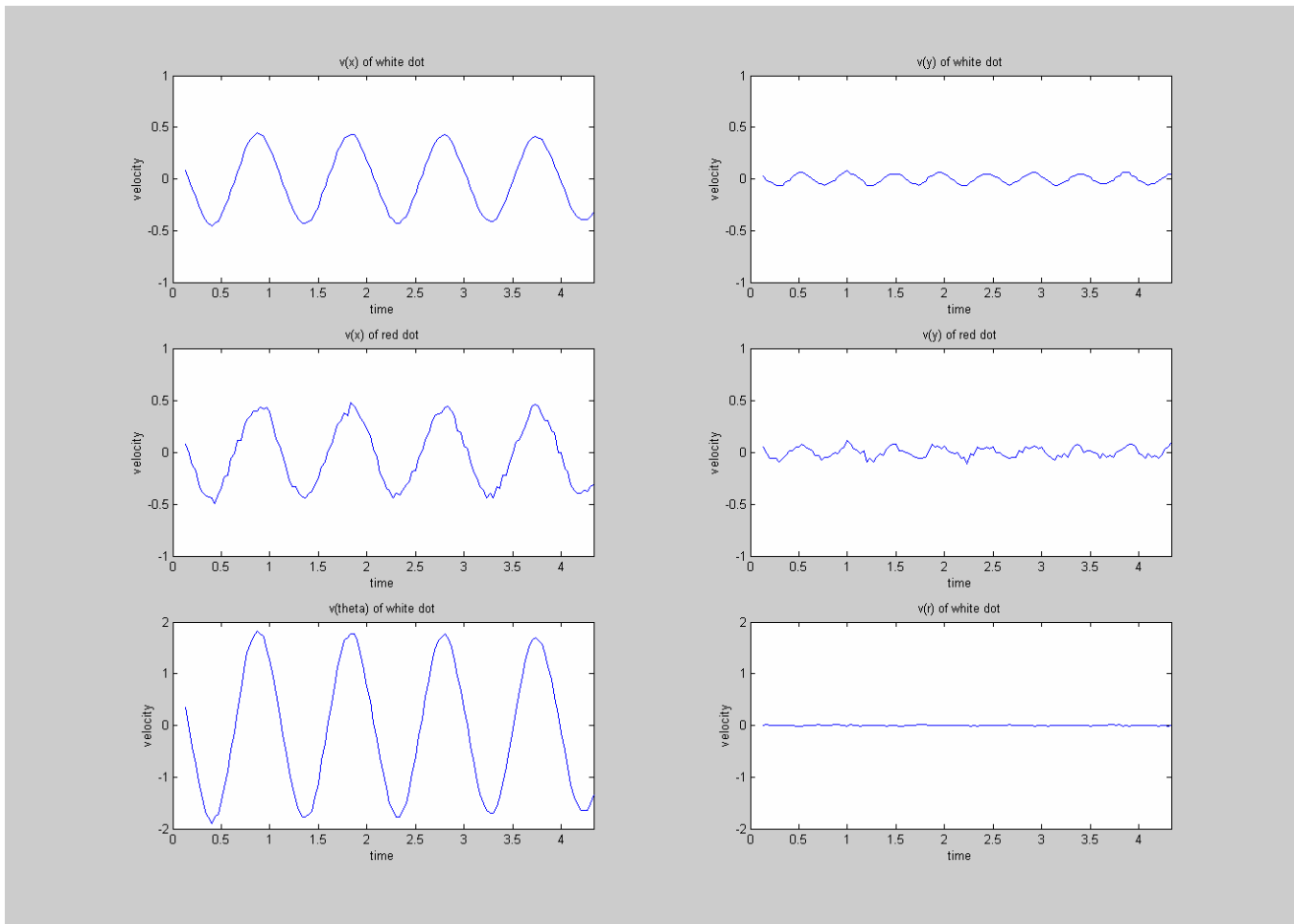
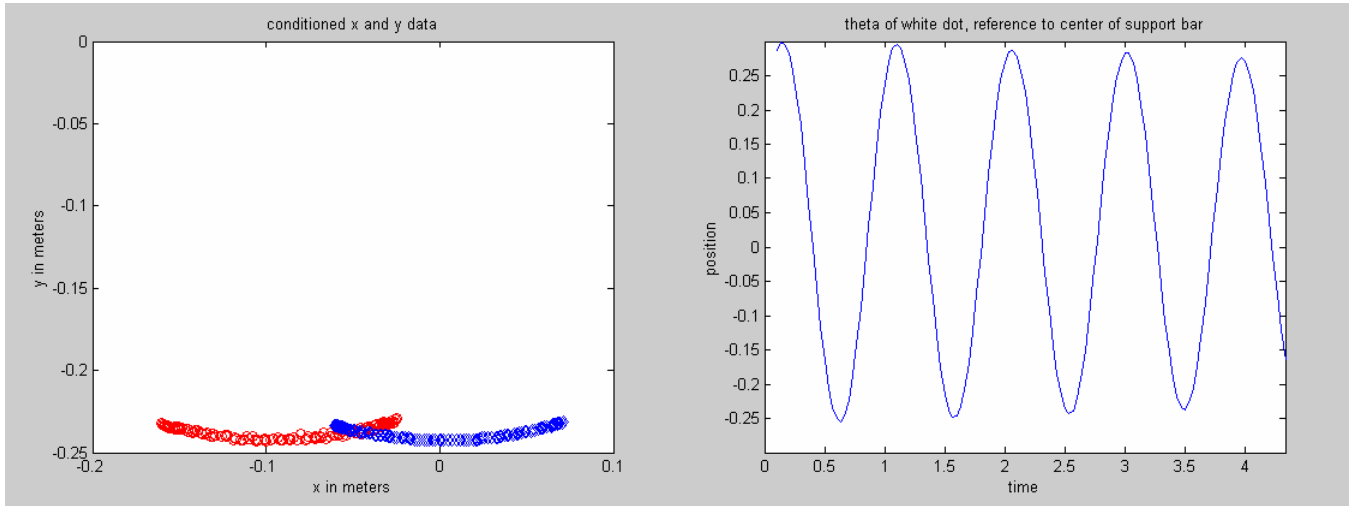


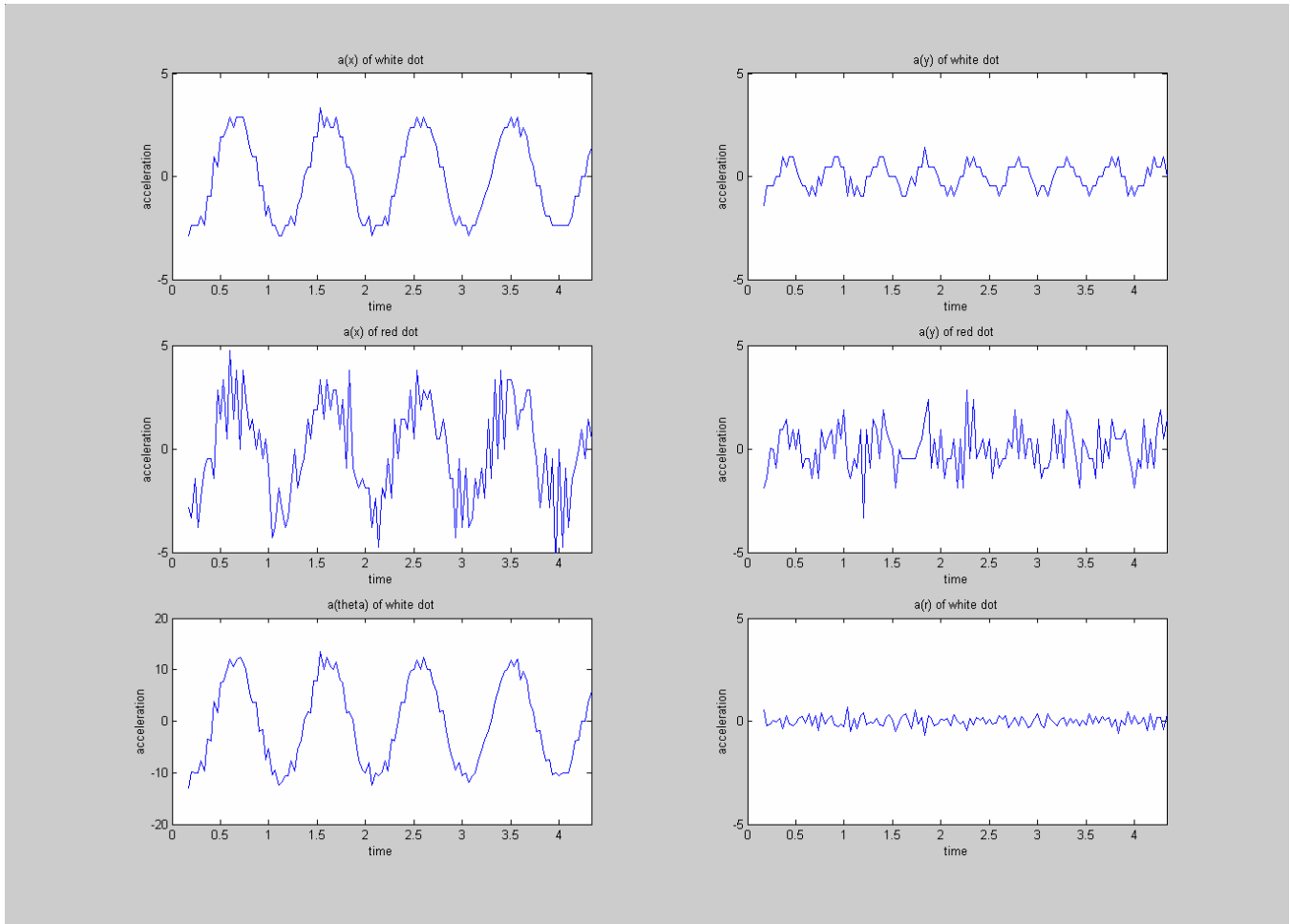
Lab 3: Rod Pendulum: Rigid Body Dynamics

Solutions

Task 0: One can find the COM by assuming evenly distributed mass and measuring geometrically, or more accurately, finding the point at which the mass balances perfectly.

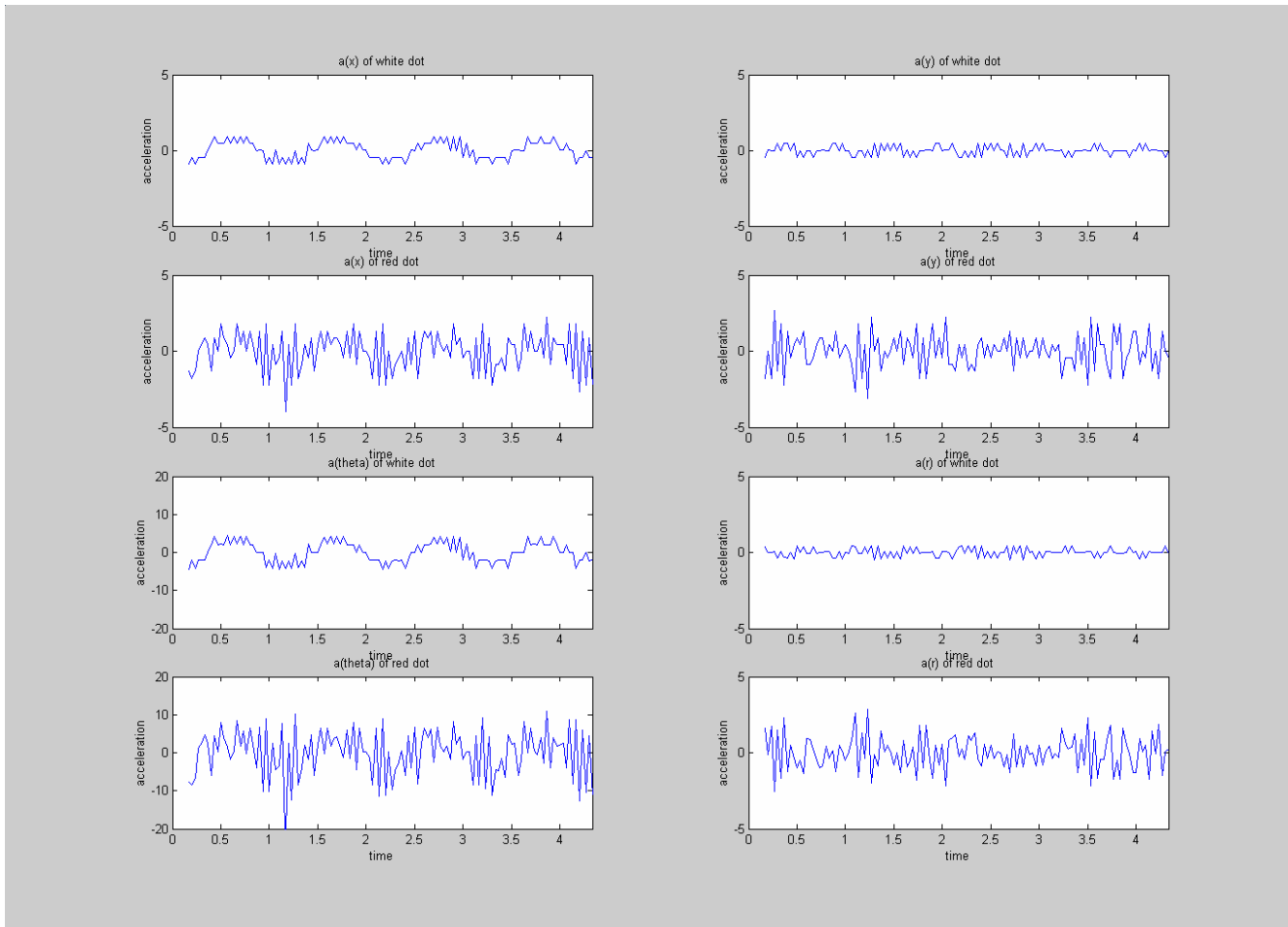
a=b:





From the prelab, we determined frequency to be $\sqrt{g/l}$. This means that theoretically, the frequency in the first configuration should have been 6.36 rad/sec (quite close).

$a < b$



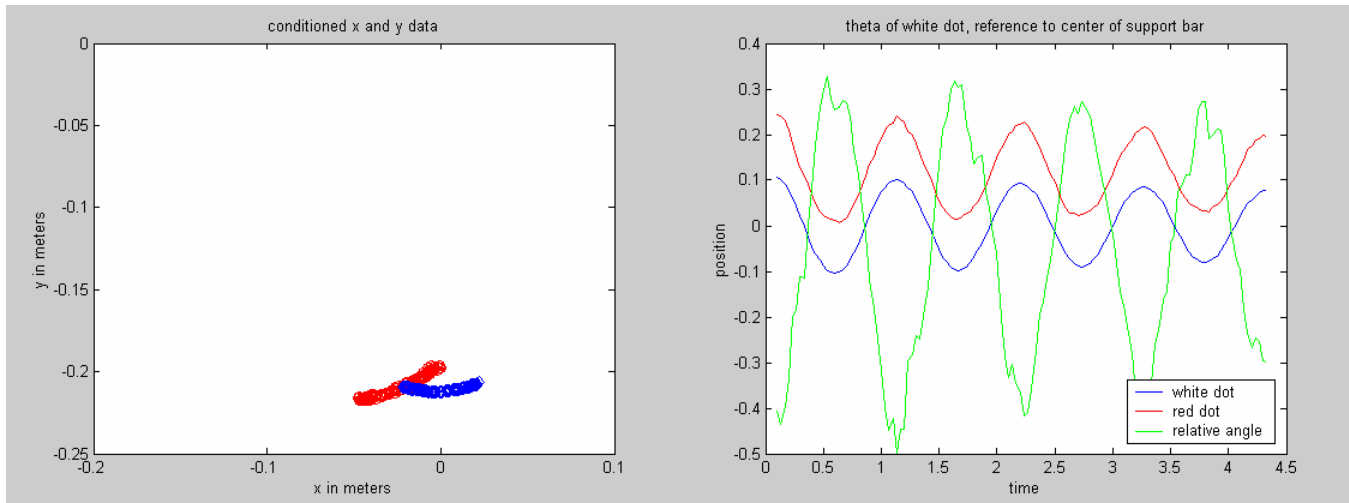
b) the markers at the tethers are $|a/2|$ away from the center of mass marker, and at an angle specified in the prelab through the constraint equations

$$L \cos \theta_1 + a \cos \theta + L \cos \theta_2 = b;$$

and

$$L \sin \theta_1 - a \sin \theta = L \sin \theta_2;$$

Also, the red dot at the tether pivot is constrained to move in a circular path defined by the tether arm.



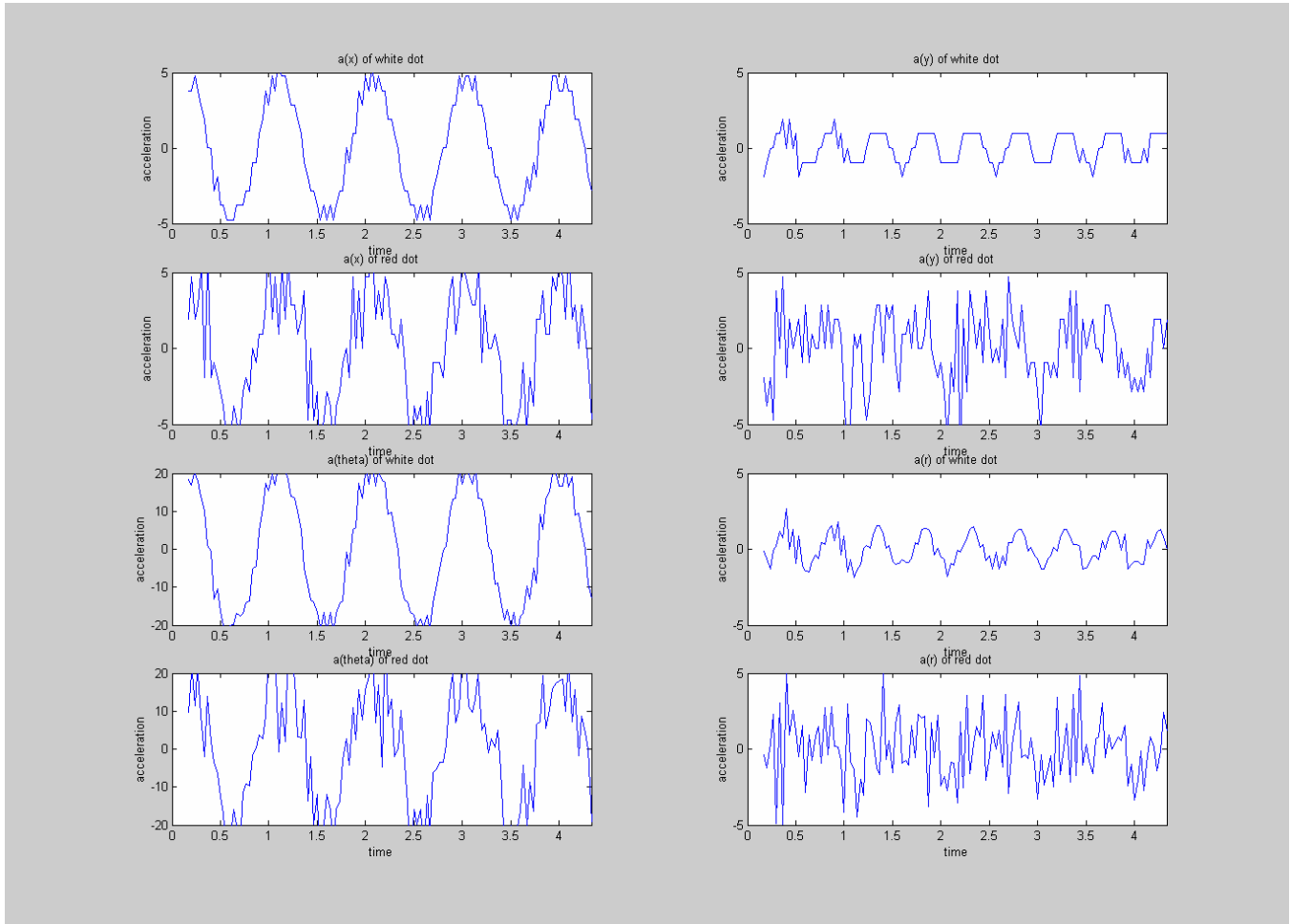
c) The natural frequency would be given by the θ of the white dot, acting as a simple pendulum with effective length given by $\sqrt{L^2 - (b/2 - a/2)^2}$. From the plot above, this appears to be approx 1 Hz, or 6.28 rad/sec. In the prelab, ω_n theoretically is given by

$$\omega_n = \sqrt{\frac{6 * g a \sqrt{L^2 - \left(\frac{b}{2} - \frac{a}{2}\right)^2}}{L^2 * (b - a)}}, \text{ where } b = 20.25\text{cm}, a = 2.5\text{cm}, L = 23\text{cm}, \text{ and } g = 9.81\text{m/s}^2.$$

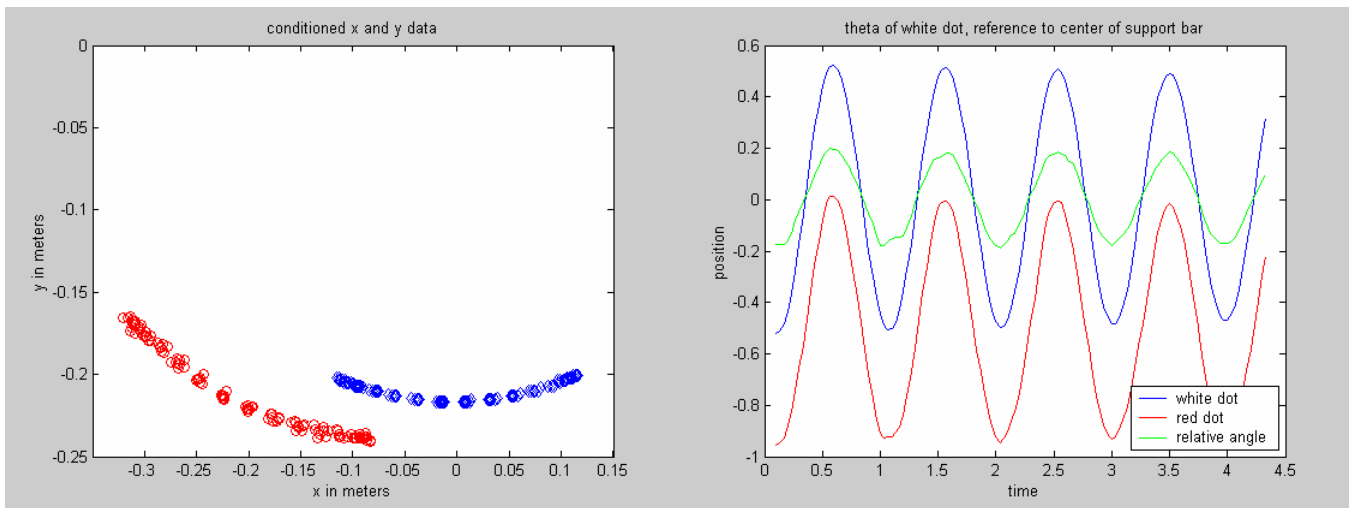
which in this case, gives us 5.76 rad/sec, not too bad.

d) $\theta(t)$ was calculated from the data as shown in the plot on the right above. Precisely, it was determined as $\text{atan2}((y_w - y_r), (x_w - x_r))$, as you can see in the code appended at the end of this assignment. It has the same frequency as the overall motion if the center of mass, as it should, given geometric constraints, although it has a much greater amplitude of motion.

$b < a$



b) again the red dot is constrained to move in a circle determined by the length of the tether arm. then the magnitude of the distance from the red dot to the white dot (at the center of mass) is $a/2$, and the angle is given by the constraint equations found in the prelab.



c) The natural frequency of this system is again approximately 1Hz, or 6.28 rad/sec, which makes sense, since the length of the pendulum arms didn't change from the last setup to this one. Using the natural frequency calculation from the prelab again,

$$\omega_n = \sqrt{\frac{6 * gb \sqrt{L^2 - (\frac{a}{2} - \frac{b}{2})^2}}{L^2 * (a - b)}}, \text{ with } b = 5\text{cm}, a = 20.5\text{cm}, L = 23\text{cm}, \text{ and } g = 9.81\text{m/s}^2.$$

This gives us $\omega_n = 8.8162$ rad/sec. This doesn't match our predicted value, but remember, we used a small angle approximation to find this value, and we are clearly not swinging at a small angle according to the plot above.

d) $\theta(t)$ is displayed above on the right, and was found the same way as in task 1. Here we see a minimization of the amplitude of θ of the bar about its COM compared to the amplitude of the rotation of the COM about its imaginary pivot point.

Additional Questions

- 1) No, linear momentum is not conserved in this system, as the velocity is continually changing as a function of time.
- 2) Yes, the bar pendulum is rotating in the cases where a is not equal to b , as seen in the plots of θ . This is due to net torques on the bar as the system is in action.
- 3) Torque is equal to $I_{\text{bar}} * \theta''$
- 4) No, angular momentum is not conserved, for the same reason given in 1).

attached please find the code that I used for analysis. looks a lot like that from lab 4.

```
twocolor;          % run twocolor to analyze the data

%----- Data Conditioning -----%
a = 0.205;         %measured separation of white/red dots at bar
b = 0.05;         %measured separation of white/red dots at bar
l = 0.23;         %measured length of the tether, from pivot to pivot.
leff = sqrt(l^2 - (b/2 - a/2)^2); %vertical length from center of support to center of
bar.

%scale data to give accurate meter conversion:
diffx = (xw-xr);
diffx = sum(diffx)/length(diffx);

xrr = xr*a/diffx;
xww = xw*a/diffx;
yrr = yr*a/diffx;
yww = yw*a/diffx;

%now average data to smooth it out
[tav,xrr] = average(tw,xrr,3);
[tav,xww] = average(tw,xww,3);
[tav,yrr] = average(tw,yrr,3);
[tav,yww] = average(tw,yww,3);
```

```

%now recenter both red and white data so that (0,0) is at center of each
%dot's arc, and up from the bottom of white dot's arc by length of the tether.
midwx = min(xww) + (max(xww)-min(xww))/2;
xww = xww - midwx;
xrr = xrr - midwx;

midwy = min(yww);
yww = yww - midwy - leff;
yrr = yrr - midwy - leff;

%plot new figure
figure(1)
plot(xrr,yrr,'ro',xww,yww,'bd'), axis([-0.35 0.15 -0.25 0]),
xlabel('x in meters'),ylabel('y in meters'), title('conditioned x and y data')

%find the relative angle between the red dot and the white dot.
thl = atan2((yww-yrr),(xww-xrr));

%----- Task 1 - Velocities -----%
%recenter red dot now, as we did with the white dot.
xrr = xrr + a - (max(xrr)-min(xrr))/2;

figure(6)
plot(xrr,yrr,'ro',xww,yww,'bd')

thw = atan2(yww,xww)+pi/2;
rww = sqrt(xww.^2 + yww.^2);

thr = atan2(yrr,xrr)+pi/2;
rrr = sqrt(xrr.^2 + yrr.^2);

figure(2)
plot(tav,thw,'b',tav,thr,'r',tav,thl,'g'),xlabel('time'),ylabel('position'),title('t
heta of white dot, reference to center of support bar')

[dt, dxww] = derivative(tav,xww);
[dt, dyww] = derivative(tav,yww);
[dt, dxrr] = derivative(tav,xrr);
[dt, dyrr] = derivative(tav,yrr);
[dt, dthw] = derivative(tav,thw);
[dt, drww] = derivative(tav,rww);
[dt, dthr] = derivative(tav,thr);
[dt, drrr] = derivative(tav,rrr);
[dt, dthl] = derivative(tav,thl);

figure(3)
subplot(4,2,1)
plot(dt,dxww),xlabel('time'),ylabel('velocity'),title('v(x) of white dot')
axis([0 dt(end) -1 1])
subplot(4,2,2)
plot(dt,dyww),xlabel('time'),ylabel('velocity'),title('v(y) of white dot')
axis([0 dt(end) -1 1])
subplot(4,2,3)
plot(dt,dxrr),xlabel('time'),ylabel('velocity'),title('v(x) of red dot')
axis([0 dt(end) -1 1])
subplot(4,2,4)
plot(dt,dyrr),xlabel('time'),ylabel('velocity'),title('v(y) of red dot')
axis([0 dt(end) -1 1])
subplot(4,2,5)
plot(dt,dthw),xlabel('time'),ylabel('velocity'),title('v(theta) of white dot')
axis([0 dt(end) -2 2])
subplot(4,2,6)

```

```

plot(dt,drww),xlabel('time'),ylabel('velocity'),title('v(r) of white dot')
axis([0 dt(end) -2 2])
subplot(4,2,7)
plot(dt,dthr),xlabel('time'),ylabel('velocity'),title('v(theta) of red dot')
axis([0 dt(end) -2 2])
subplot(4,2,8)
plot(dt,drrr),xlabel('time'),ylabel('velocity'),title('v(r) of red dot')
axis([0 dt(end) -2 2])

figure(4)
plot(dt,dthl),xlabel('time'),ylabel('velocity'),title('relative velocity of white
and red dots')

```

```

%----- Task 2 - Accelerations -----%

```

```

[ddt, ddxww] = derivative(dt,dxww);
[ddt, ddyww] = derivative(dt,dyww);
[ddt, ddxrr] = derivative(dt,dxrr);
[ddt, ddyrr] = derivative(dt,dyrr);
[ddt, ddthw] = derivative(dt,dthw);
[ddt, ddrww] = derivative(dt,drww);
[ddt, ddthr] = derivative(dt,dthr);
[ddt, ddrrr] = derivative(dt,drrr);

figure(5)
subplot(4,2,1)
plot(ddt,ddxww),xlabel('time'),ylabel('acceleration'),title('a(x) of white dot')
axis([0 ddt(end) -5 5])
subplot(4,2,2)
plot(ddt,ddyww),xlabel('time'),ylabel('acceleration'),title('a(y) of white dot')
axis([0 ddt(end) -5 5])
subplot(4,2,3)
plot(ddt,ddxrr),xlabel('time'),ylabel('acceleration'),title('a(x) of red dot')
axis([0 ddt(end) -5 5])
subplot(4,2,4)
plot(ddt,ddyrr),xlabel('time'),ylabel('acceleration'),title('a(y) of red dot')
axis([0 ddt(end) -5 5])
subplot(4,2,5)
plot(ddt,ddthw),xlabel('time'),ylabel('acceleration'),title('a(theta) of white dot')
axis([0 ddt(end) -20 20])
subplot(4,2,6)
plot(ddt,ddrww),xlabel('time'),ylabel('acceleration'),title('a(r) of white dot')
axis([0 ddt(end) -5 5])
subplot(4,2,7)
plot(ddt,ddthr),xlabel('time'),ylabel('acceleration'),title('a(theta) of red dot')
axis([0 ddt(end) -20 20])
subplot(4,2,8)
plot(ddt,ddrrr),xlabel('time'),ylabel('acceleration'),title('a(r) of red dot')
axis([0 ddt(end) -5 5])

```