Vorticity in the ocean...

2.011 Lecture

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Big whirls have little whirls…

"Big whirls have little whirls,
That feed on their velocity,
and little whirls have lesser whirls,
and so on to viscosity."

-Lewis F. Richardson (1920)

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What is a vortex?
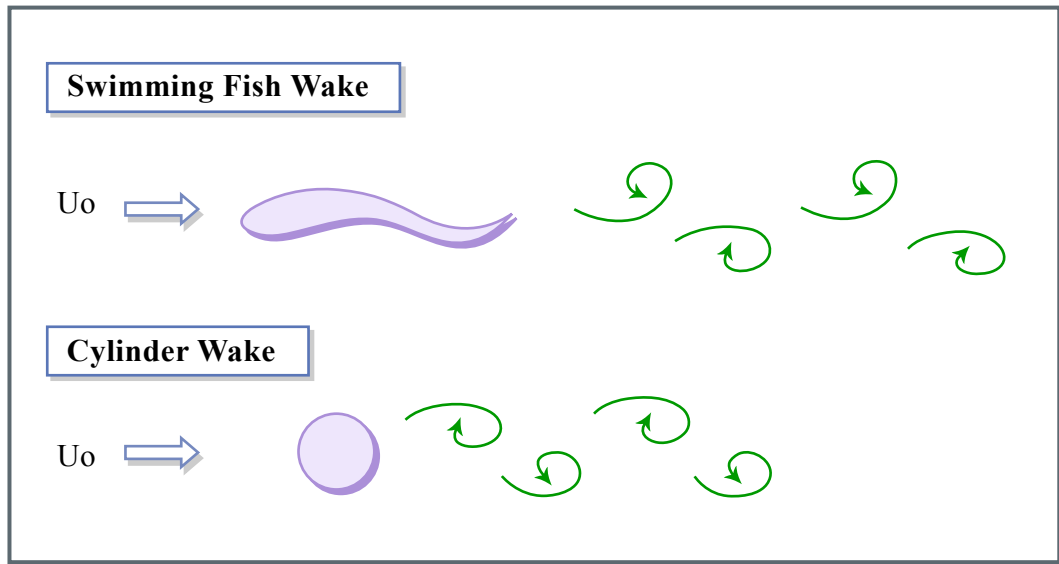
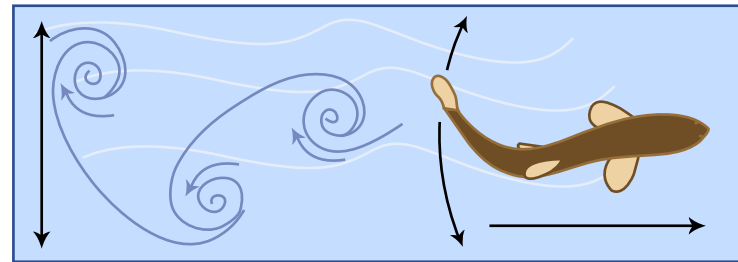- Swirling/rotating region of fluid

Figure by MIT OCW.

Figure by MIT OCW.

Tornados and Water Spouts

Courtesy of U.S. National Oceanic and Atmospheric Administration (NOAA).
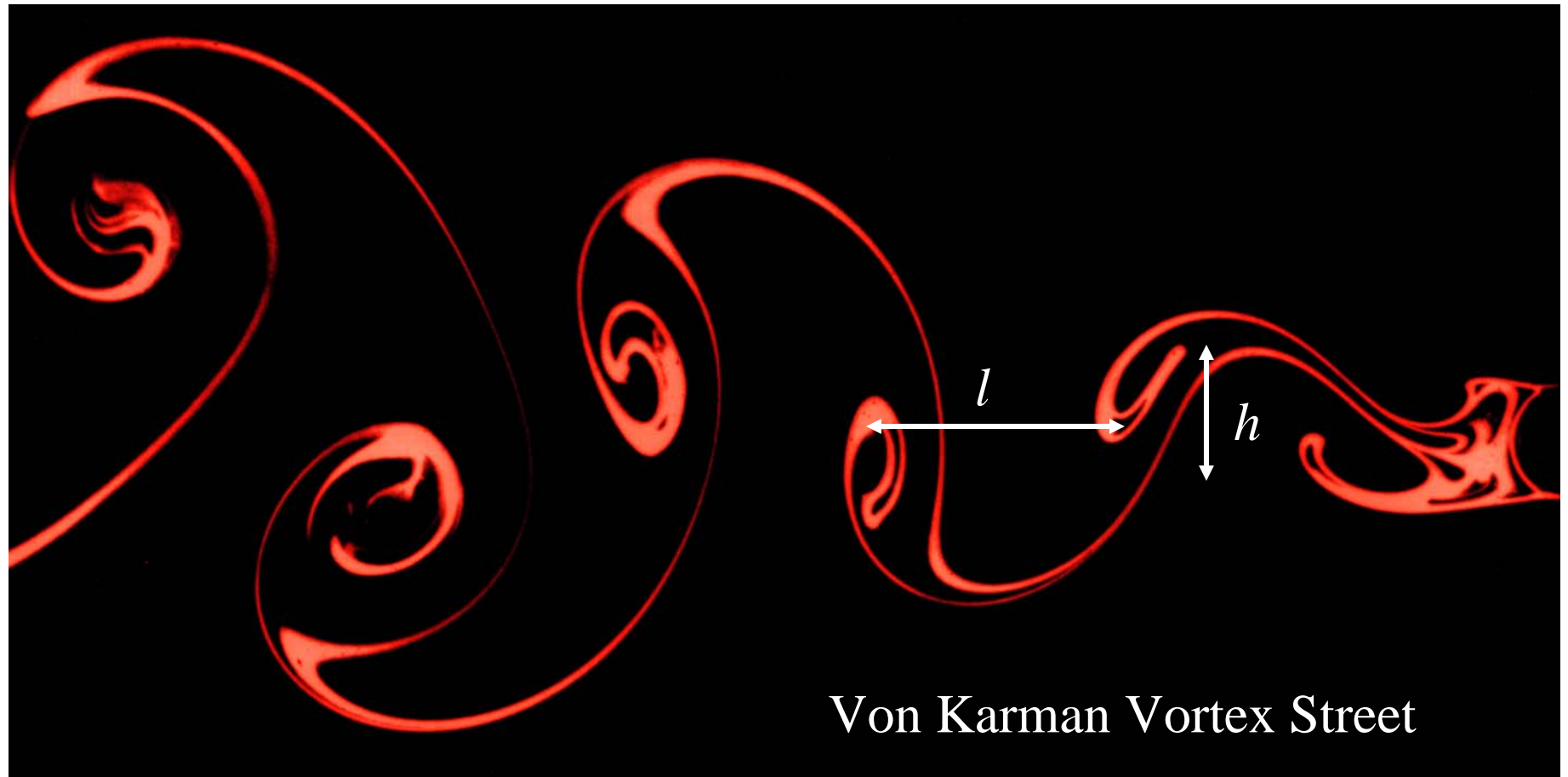
Vortex Rings and Wake Morphology

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Vortex Shedding

- When the flow separates vortices are shed in the wake.

- Vortices are small "eddies" that result in a force on the body.

- Recall vorticity is defined as the curl of the velocity field:

$$\vec{\omega} = \nabla \times \vec{V}$$

Classical Vortex Shedding

Von Karman Vortex Street

Alternately shed opposite signed vortices

Vortex shedding results from a wake instability

Sequence of six photos removed for copyright reasons.

Diagrams of vortex structure due to Flapping (birds, fish), Undulating (eels), and Paddling (frogs) - removed for copyright reasons.

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Governing Equations
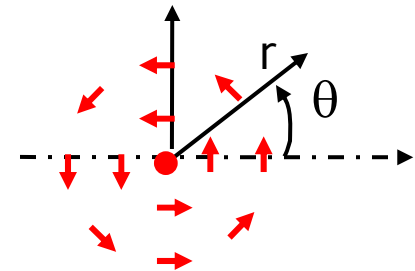
- Continuity equation (conservation of mass)

$$\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- Navier-Stokes equations (F= ma)

$$\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\mathbf{V} + \mathbf{g} + \vec{F}_{coriolis}$$

- Vorticity:

$$\boxed{\omega \equiv \nabla \times \mathbf{V}}$$

Irrotational Fluid

- For *irrotational flow* the *average angular velocity of every fluid particle is everywhere zero*.

- This does not mean that the flow cannot be rotating in general, relative to some reference frame – just that the particles themselves are not rotating.
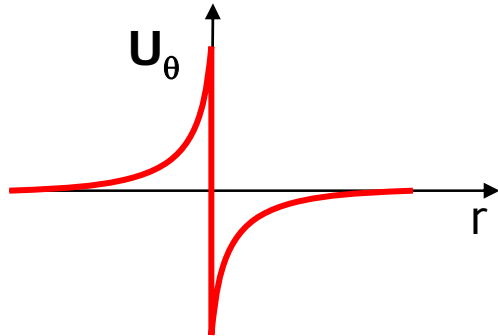
Vorticity
$$\omega \equiv \nabla \times \mathbf{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = 0$$

$$\omega = \hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \hat{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

Vortex Velocity

- Looking at the curl of the velocity, for a two-dimensional flow field, where *x* is the horizontal axis and *y* is the vertical axis, we can show that equation simply reduces to

$$\omega = \hat{k}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) = 0$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

U$_\theta$

r

- Plot of velocity as a function of radius from the vortex center. At the core of the potential (inviscid) vortex the velocity blows up to infinity and is thus considered a singularity.

- This is not true in a real, or viscous, fluid. Viscosity prevents the fluid velocity from becoming infinite at the vortex core and causes the core rotate as a solid body. The flow in this core region is no longer considered irrotational.

Fish Vortex Wake

Figures removed for copyright reasons.

Source: paper by Nauen and Lauder, 2002.

Turbulence

- Turbulence is the irregular, random component of fluid motion.
- Its spatial scales are usually the smallest scales of the flow (from 0.01 to 10-meter overturning motions that result in vertical transport and mixing)
- Our conceptual framework for understanding turbulence is largely statistical.
- Turbulent motions in a stratified fluid (layered fluid of different densities) mix fluid parcels from different parts of the flow, thereby enhancing mixing across concentration gradients by molecular diffusion.
- Oceanographers want to know the end result: the rate at which the ocean is mixed.
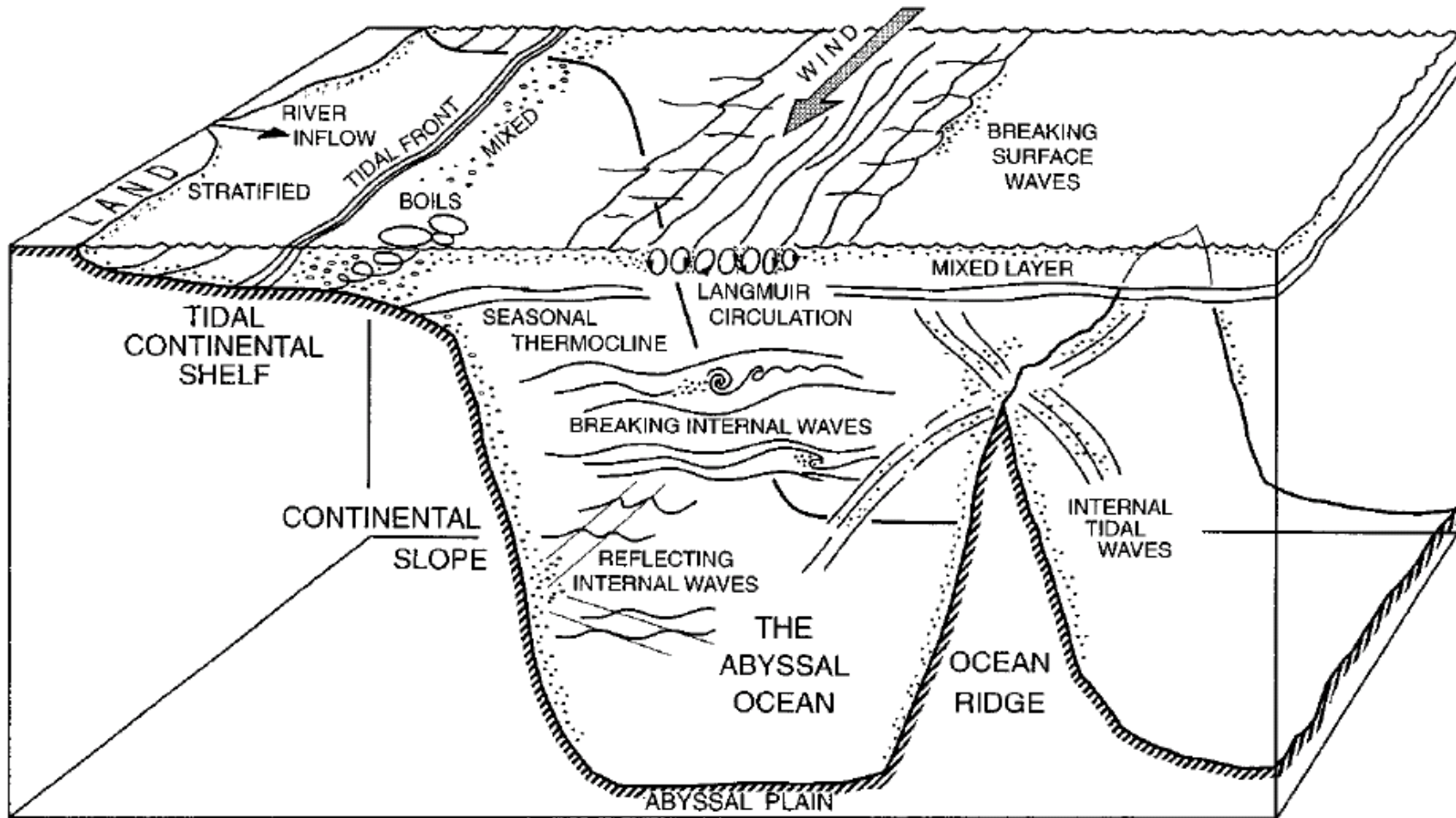
Sources of Turbulence

Figure 1 Sketch (not to scale) showing some of the features mentioned in text that are related to ocean turbulence. Regions of strong turbulence are stippled.

Courtesy of Annual Reviews. Used with permission.

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Evidence of "Coherent Structures" in the Ocean

A "coherent structure" may be thought of as a shape or form in a turbulent fluid flow that persists a long time relative to it's own period of internal circulation.
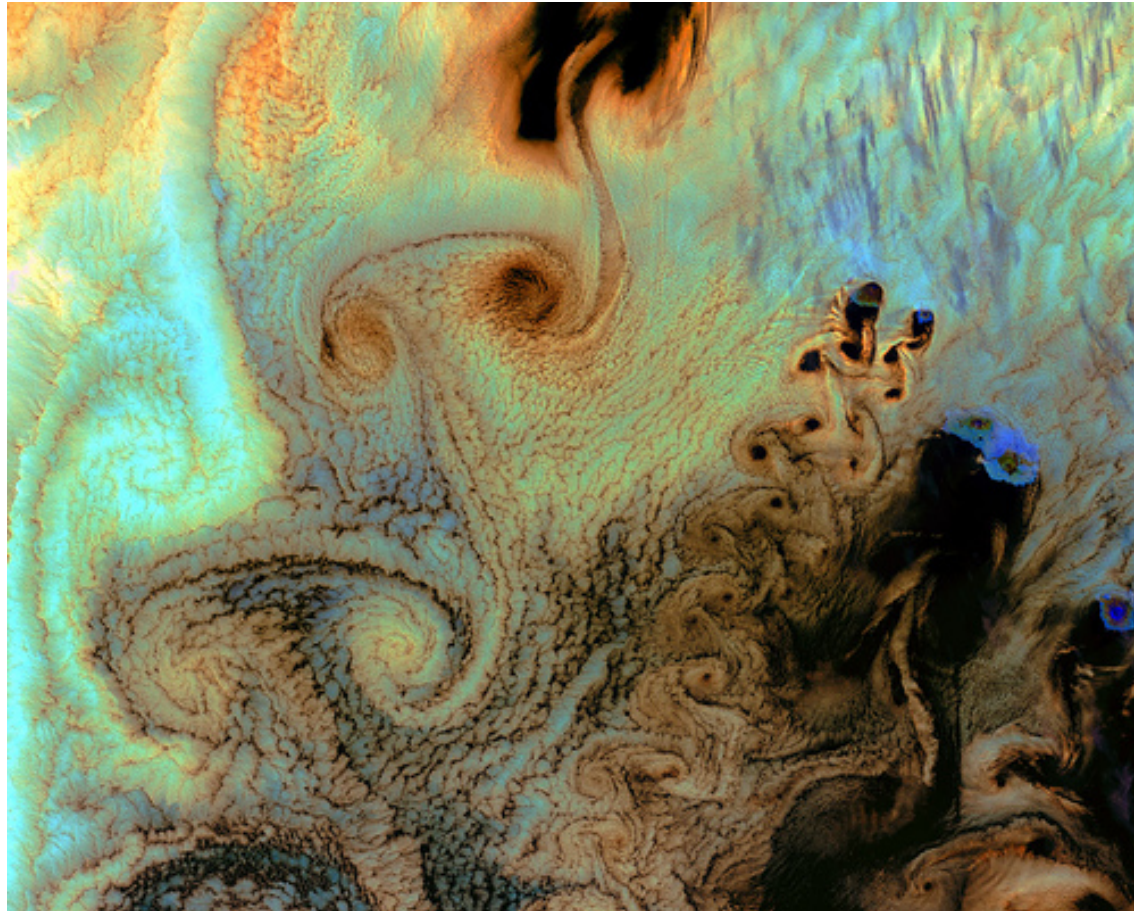
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Gulf Stream - Spiral Eddies
Greek Islands - Spiral Eddies and Wakes
Gulf Stream - Shear zone and associated cold core eddies, shown by plankton bloom

Vorticity in the Atmosphere

Karman vortices over the Aleutian islands in Alaska, viewed in infrared bands.

Courtesy of U.S. Geological Survey (USGS).

Regions of strong vorticity are associated with upwelling and downwelling and high plankton activity

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Large Eddies

- The technique (subsequently termed the large-eddy technique) which will be implemented here originates in simple scaling analysis of the turbulent kinetic energy equation (Taylor 1935: Tennekes and Lumley 1972).

- If *q' and λ are respectively velocity and length scales characteristic of the largest energy-containing eddies of a turbulent flow, $\tau \sim \lambda / q'$ is the associated large-eddy time scale and*

$$\varepsilon \sim \frac{(q')^3}{\lambda} \sim \frac{(q')^2}{\tau}$$
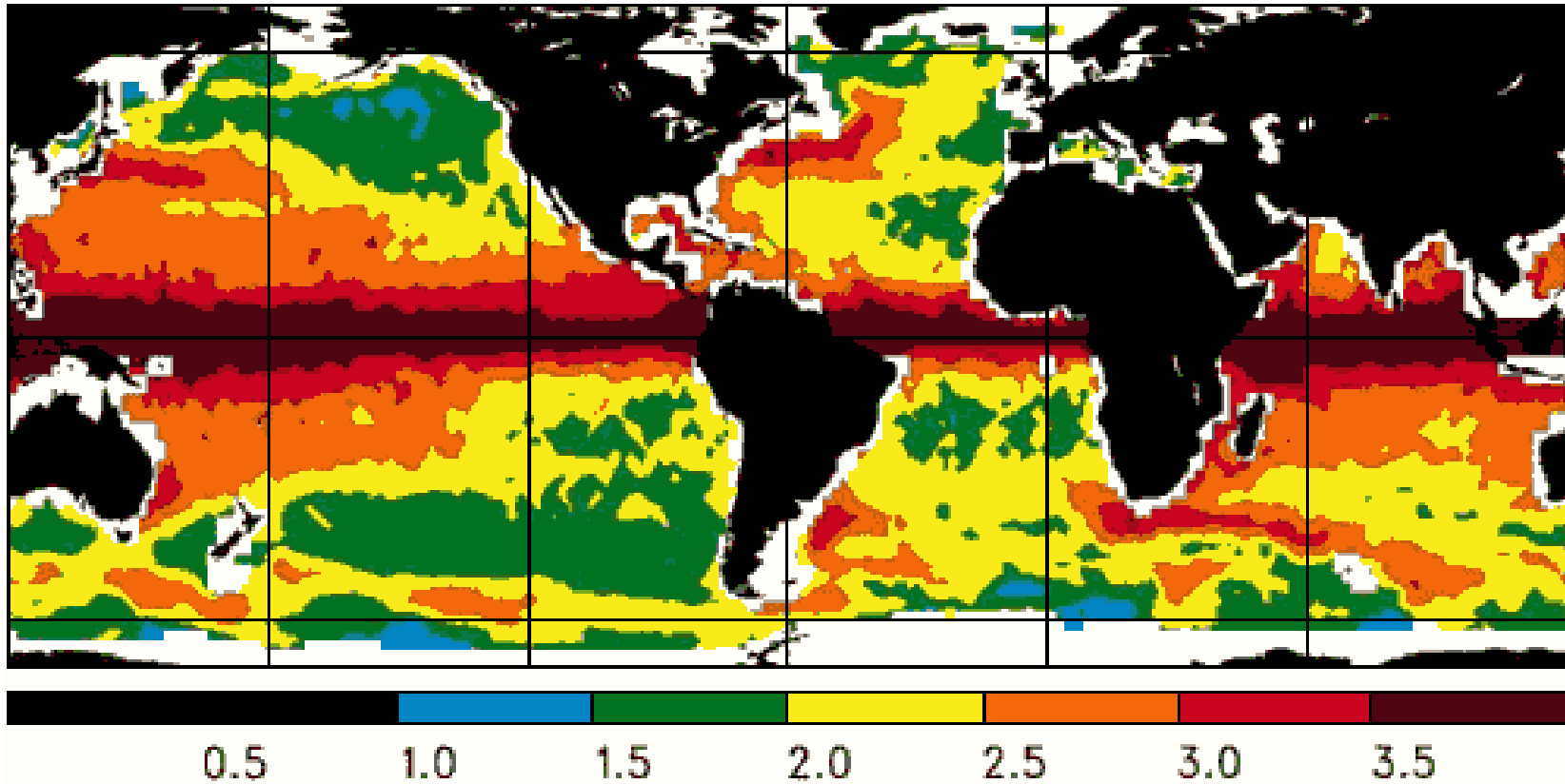
Eddy kinetic energy, inferred from three years of TOPEX data. (Figure 2 in Stammer 1997. *J. Phys. Ocean* **27**, 1743.)

Turbulent measurements

- Fundamental quantities used in the measurement of oceanic turbulence are:

    - ε == the rate of dissipation of kinetic energy per unit mass.
        - It tells us the rate at which energy is converted to heat by friction.

    - χ == the rate of destruction of temperature variance, i.e. the rate at which temperature gradients are smoothed by molecular diffusion.
    - **N**, the buoyancy frequency, provides a time scale for many aspects of stratified flow.
    - Kolmogorov scale, $\eta = \left(\upsilon^3 / \varepsilon \right)^{1/4}$ is an important spatial scale
        - It is the smallest scale on which velocity gradients can exist in the face of viscous smoothing. (ν is the kinematic viscosity)
    - v_k == friction velocity (near the wall)
        - $v_k = (\tau/\rho)^{1/2}$, τ being the surface stress and ρ the fluid's density.
    - κ == vonKarman's constant ($\kappa \sim 0.4$).

Ocean Mixing

- But to understand the mixing, we have to deal with the messiness of turbulence.
- Turbulent motions are rapidly dissipated in the absence of an energy source
- It is critical that we identify and quantify the sources of turbulence.
- Understanding mixing is especially important for ocean models designed to predict global circulation, climate change, pollutant dispersal and primary productivity,
- These models can provide reliable predictions only when we have the capability to quantify the subgrid-scale effects of turbulence.

Turbulent Mixing

- Without turbulence and the mixing it causes, we would not have the same ocean that we do now, nor indeed the same climate.
- Turbulent mixing brings nutrients into the surface layer from below so that plankton can grow.
- Turbulence near the surface, driven by surface winds and cooling, transmits heat in and out of the ocean to create the reservoir of heat that governs climate.
- It is turbulence that diffuses the permanent pycnocline separating the cold bottom waters of polar origin from the atmosphere-connected upper ocean, either directly, or indirectly by local mixing followed by distribution along isopycnals (surfaces of constant density).
- Turbulence in the bottom layer affects the deposition, resuspension and movement of sediments.
- Turbulence creates micro-environments for the small creatures that form the basis for life in the oceans.